UNCLASSIFIED

AD NUMBER

AD444364

NEW LIMITATION CHANGE

TO

Approved for public release, distribution unlimited

FROM

Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; 01 AUG 1964. Other requests shall be referred to Ballistic Systems Division, Norton AFB, CA.

AUTHORITY

SSD ltr dtd 9 Mar 1965

THIS PAGE IS UNCLASSIFIED

TRW SPACE TECHNOLOGY LABORATORIES

THOMPSON RAMO WOOLDRIDGE INC.

ONE SPACE PARK • REDONDO BEACH, CALIFORNIA

Total Pages: 107

NUMERICAL ANALYSIS OF SYSTEM AVAILABILITY AND OF PARAMETER ESTIMATION METHODS

Prepared by

W. J. Howard
N. Kaufman

TRW SPACE TECHNOLOGY LABORATORIES
Redondo Beach, California

6101-8161-TU001

Contract No. AF 04(694)-524

August 1963

(Revised August 1, 1964)

Prepared for

HQ BALLISTIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
Air Force Unit Post Office, Redondo Beach, California

Prepared for
HQ BALLISTIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND, USAF
Under Contract No. AF 04(694)-524

Prepared: W. J. Howard

W. J. Howard

N. Kaufman

N. Kaufman

Approved: J. M. Schwarzbach

J. M. Schwarzbach
Manager,
Systems Engineering Department
Atlas Program Office

TRW SPACE TECHNOLOGY LABORATORIES
One Space Park . Redondo Beach, California

ACKNOWLEDGEMENT

The authors wish to thank A. J. Monroe for useful discussions and criticism, and for the model (References 1 and 2) which is subjected to numerical analysis in Part III and Appendix B; R. G. Schluter and S. Lass for developing the computer programs used in the analysis; and J. D. Brooks, J. W. Green, D. A. Pope, and J. F. Zancanaro for investigating the functions arising in connection with one of the methods of parameter estimation and for devising the computer search routines described in Reference 7.

ABSTRACT

This document describes two problem areas concerning systems subject to periodic checkout, which were investigated with the aid of a computer. The first part (Parameter Estimation) summarizes the results of a Monte Carlo analysis to determine the feasibility and accuracy of measuring system failure rates, checkout error probabilities, and repair effectiveness from the numbers of systems passing and failing in three consecutive checkouts. The second part (Availability Analysis) describes, mainly through a series of graphs, the quantitative variation of system availability with a number of operational and maintenance parameters representing time durations of standby, checkout, and repair, failure rates during standby and checkout, repair effectiveness, decision errors during checkout, and test coverage.

PREFACE TO THE REVISED PRINTING

This edition corrects the equation for P_{AR} given in Figure 4, page 28. Figures B-1 to B-46, in Appendix B, have been revised to correspond to the corrected equation. The changes in the figures are, for the most part, relatively minor, and none of the conclusions of the report require modification. (Some of the changes in the figures are only apparent, due to a change in scale of the abscissa).

A typographical error in the equation for P_r (PFP) on page 12 has also been corrected (thanks to H. Jaffe).

CONTENTS

CONTENTS (Continued)

ILLUSTRATIONS

ILLUSTRATIONS (Continued)

ILLUSTRATIONS (Continued)

TABLES

I. INTRODUCTION AND SUMMARY

A. Introduction

One of the major factors contributing to the effectiveness of the Atlas ballistic missile weapon system is availability or alert readiness. Mathematical models relating this factor to the hardware, procedures and personnel aspects of the system have been developed and are described in References 1, 2 and 5. Concurrently with the development of these models, there has been an investigation into the question of how to estimate or measure the parameters used in the availability equations. This question was first considered in detail in Reference 3, which describes two possible methods of parameter estimation for periodically checked systems. Another method is described in Reference 6. Parameter estimation for continuously monitored systems is considered in Reference 4.

The significance of the parameter estimation problem for the alert readiness models arises from the following conditions:

(1) The models include the possible effects of checkout error, incomplete test coverage, and imperfect repair; these effects cannot be measured directly by means of routine failure and maintenance data, as the data itself includes these errors to an unknown extent.

(2) Although it is theoretically possible to initiate a comprehensive program of failure analysis and operational surveillance to provide the kind of data required, there is no data of this sort presently available, nor is such a program contemplated. Moreover, the cost of such a program could well be prohibitive.

For these reasons, the possibility of using existing or easily obtainable data to estimate the model parameters by inferential or indirect methods has been analyzed in considerable detail. Reference 3 describes two basic methods which could be used for indirect parameter estimation. In order to check the accuracy and potential usefulness of these methods, a series of Monte Carlo trials was run on the

7090 computer. The trials were conducted using various input parameter values and sample sizes, and estimated parameter values were automatically computed using the estimating equations, in the same manner as would be the case if actual field data were available. Comparison of the estimated with the true parameter values allows an assessment of the effectiveness of the estimating procedures. Section II discusses the two estimation methods, the corresponding computer programs, and the results obtained to date.

When the attempt is made to account for most of the potentially important factors affecting alert readiness, the resulting mathematical models yield complicated equations for availability, with numerous parameters. A general expression for the alert readiness of a periodically checked system is given in Reference 2 (Equation 54, p. 10). This equation includes all parameters used in the mathematical models to date to characterize periodically checked systems. The resulting equation is sufficiently complex that a computer program was developed to investigate the effects of the various parameters on system availability. To simplify presentation of results, a plotter was connected to the computer output. Section III discusses the equation and presents the numerical results obtained.

Because of the volume of data summarized for the Monte Carlo trials and the computer analysis of availability, the tables and graphs have been placed in Appendices A and B, respectively. To aid in interpreting the data, as well as to aid in reading the report, a fold-out list of symbol definitions used in the Monte Carlo trials is provided at the end of Appendix A, and a similar fold-out list of symbols used in the availability analysis appears at the end of Appendix B. There is some difference in notation for the two lists, as the computer programs originated from different projects, and as it is believed desirable to to indicate the symbols as they actually appear in the existing programs.

For the reader interested only in the main results of the computer analyses, a summary follows.

B. Summary

This document reports the results of two projects pertaining to system availability which required extensive numerical analysis.

The first project involved a series of Monte Carlo runs in which the time sequence of events of individual units passing through repair, standby, and checkout was simulated, the objective being to determine if or to what degree of accuracy the system failure rate, repair effectiveness, and error frequency during checkout could be estimated from the checkout results alone. The second project was a parametric analysis of system availability using a curve plotting machine connected to the computer output to aid in revealing the sensitivity of availability to the various system operational and maintenance parameters. Parameters included in the analysis were standby, checkout, and repair periods, system failure rate during standby and checkout, repair effectiveness, test coverage, and decision errors during checkout.

The major results of these projects are presented in tabular and graphical form in the appendices. Appendix A is a series of tables describing the parameter estimates from the Monte Carlo runs. Appendix B is a representative selection of graphs obtained in the parametric study of availability. While the tables and graphs speak for themselves, some of the more obvious results are summarized below.

1. <u>Parameter Estimation</u>

Of the two methods of indirect parameter estimation for periodically monitored systems described in Reference 3, the Variable Standby Time Method and the Multiple Checkout Method, only the latter method was subjected to a relatively thorough numerical analysis via Monte Carlo runs on a computer. A computer search routine, described in Reference 7, was developed for processing data applicable to the Variable Standby Time Method, but a thorough numerical test was not performed for this method. Some small-scale sample results are reported in Reference 7. The numerical results described in this report, and summarized here, apply therefore strictly to the Multiple Checkout Method.

Briefly, this method prescribes that a group of systems enter a standby period, followed by three consecutive checkouts. Whether the units are operating or not during standby is immaterial,

but the standby mode (which may be of zero length) is assumed the same for all units. Based exclusively on the number of units passing or failing on each of the three checkouts, the problem is to determine, if possible, the values of the unit survival probabilities during standby (p_s) and checkout (p_{c_1} and p_{c_2}), the probability the unit was good upon completing repair (μ), and the probabilities of calling a good system bad (α) or a bad system good (1-E) at the point of checkout decision. It was shown in Reference 3 that, in general, the parameters α and E can be estimated, together with the products $p_{c_1} p_{c_2}$ and $\mu p_s p_{c_1}$. It was also shown that if some of the units go directly from repair into checkout, so that $p_s = 1$, and if furthermore all failures during checkout occur prior to test decision, so that $p_{c_2} = 1$, all of the remaining parameters can be estimated. (Since the product $p_{c_1} p_{c_2}$ is estimated, it is necessary only that there be an assumed relationship between p_{c_1} and p_{c_2}, rather than $p_{c_2} = 1$ specifically, for it to be possible to estimate all of the parameters.)

The parameters α and 1-E can be called "error parameters" since they describe Type I and Type II decision errors during checkout, and describe the apparent, rather than the actual, condition of the units. The other parameters can be called "state parameters," since they describe the actual condition. The Monte Carlo runs show that the accuracy of measuring the error parameters α and E depends strongly upon their values as well as the values of the other (state) parameters. The number of units (or sample size) required to form reasonable estimates varies from about 25 or 50 for E, and 100 for α, for the most favorable cases, to 10,000 or more for less favorable combinations of parameter values. There is no apparent bias in the estimates.

If only one source of error is present, the accuracy of measurement of that error increases significantly. For example, with a sample size of 500 and nominal values of the state parameters (p_s = 0.8, μ = 1.0, p_{c_1} = 0.8, p_{c_2} = 1.0), and with a true value of E = 0.9, the variance of E increases from 0.0027 for α = 0 to 0.1718 for α = 0.3, or a change in the standard deviation of estimate of almost a factor of 10. Similarly, with

the same values of the other parameters, if true $\alpha = 0.1$, the variance of $\hat{\alpha}$ is 0.0039 if $E = 1$ and 0.4182 if $E = 0.7$. These results are from 300 sample runs with 500 units simulated on each run.

The accuracy of estimation degrades rapidly if there are no true failures during checkout, as the method of estimation relies on there being a possibility of change of state of the units as they pass through the three consecutive checkouts. If no change can occur, the estimating equations degenerate, and, as with a single checkout, no estimates can be obtained for α and E unless additional information is available or additional assumptions are made.

When a series of cases was run which allowed all parameters to be estimated, the calculation of availability from these estimates gave quite reasonable values when compared to the true availability. A sample size of 1000 was used, using hand calculation on the results, and the use of smaller sample sizes should be investigated on the computer.

It was found by comparison with a sample result that confidence limits obtained on the estimates by using the normal distribution agreed closely with the percentage groupings printed-out by the computer. This method is satisfactory provided the variance is not too large.

2. Availability Analysis

The availability equation for a system subject to periodic checkout which is discussed in this report was derived in Reference 1, and is based on the assumptions that failures during standby have an exponential distribution, and standby, checkout, and repair time (if required) are of fixed duration. The parameters which this equation quantitatively relate to system availability can be grouped as follows in order to summarize:*

*Definitions of these parameters will be found on a fold-out sheet at the end of Appendix B.

(1) Duration of standby, checkout and repair periods (T_s, T_c, T_r)

(2) System failure parameters (for detectable failures only) during standby and checkout (λ_s, $p_{d_{t_{c_1}}}$, $p_{d_{t_{c_2}}}$)

(3) Error probabilities in checkout in deciding whether a system is good or bad on the basis of detectable characteristics (α, β)

(4) Partial test coverage parameters (λ_u, $p_{u_{t_c}}$, $1-\mu_1-\mu_2$)

(5) Imperfect repair parameter for detectable failures (μ_1).

A series of basic cases was run in which the independent variable was standby time (T_s), because of its importance as a parameter which is specified by maintenance policy and which can therefore be relatively easily modified as necessary. As is generally known, availability (P_{AR} for probability of alert readiness) does not usually increase or decrease monotonically with T_s. Under a given set of conditions, there is some value of T_s which maximizes P_{AR}, and this value can be considered as a partial specification of a maintenance policy.

Aside from T_s, of the above list of variables, only two, α and $p_{d_{t_{c_1}}}$, were found to have the property that P_{AR} did not always increase or decrease as the parameter varied with the other parameters held fixed. Specifically, P_{AR} decreases monotonically as the following increase: T_c, T_r, λ_s, $1-p_{d_{t_{c_2}}}$, β, λ_u, $1-p_{u_{t_c}}$, $1-\mu_1-\mu_2$, and $1-\mu_1$. The manner of variation and the interactions are too complex to summarize; however, the following trends can be noted: Certain parameters have an important influence on P_{AR} if the standby time is short, but less if the standby time is long. These are: T_c, T_r, $p_{u_{t_c}}$, $1-\mu_1-\mu_2$, and $p_{d_{t_{c_2}}}$. Other parameters are more important at longer standby times than at short. These are λ_s and λ_u. (However, even at short standby times λ_u has considerable effect.) A third group of parameters affects P_{AR} in a manner which is less dependent on standby time (but the effect is less at short times). These are β and μ_1. For large values of T_s, P_{AR} has approximately a linear relationship with all parameters except λ_s and λ_u; for these parameters, the relationship is approximately linear for small T_s. P_{AR} is approximately linear with μ_1 and $1-\mu_1-\mu_2$ for all values of T_s.

The optimum standby time was found to increase as the following increase: T_c, T_r, λ_s, λ_u, β, $1-p_{d_{tc_1}}$, $1-p_{d_{tc_2}}$, and $1-p_{u_{tc}}$. It usually increases as α increases. It remains essentially unchanged as μ_1 and $1-\mu_1-\mu_2$ vary.

The parameters α and $p_{d_{tc_1}}$ are particularly interesting since the maximum value of P_{AR} may occur at either extreme (0 or 1, as these are probabilities) or at intermediate values, depending on the other conditions. (This property of α was pointed out in Reference 8.) While the parameters α and $p_{d_{tc_1}}$ were incorporated into the model to represent sources of degradation in the form of Type I errors and early failures during checkout, respectively, they were found to operate also as "preventive maintenance" parameters. Due to the possible accumulation, with time, of undetectable failures in the system, it is best to repair/replace so-called "good" systems periodically. Both α and $p_{d_{tc_1}}$ can operate to force this result: in the case of α, "good" systems are sent to repair through error, and in the case of $p_{d_{tc_1}}$, they are sent to repair through deliberate failure just prior to the point of test decision. Whether this preventive repair/replacement is performed at every checkout, after a certain maximum number of checkouts, or never, depends on whether the optimum value of α (or of $1-p_{d_{tc_1}}$) is 1, between 0 and 1, or 0, respectively.

These remarks apply to undetectable failures occurring during standby and checkout (λ_u and $p_{u_{t_c}}$), but not to those introduced during the repair process itself $(1-\mu_1-\mu_2)$. If undetectable failures occur only during repair/replacement, this type of preventive maintenance is valueless. In addition to the values of λ_u and $p_{u_{t_c}}$, the size of T_r influences the optimum replacement period, as this is a compensating time lost from readiness. Thus, the relationship between P_{AR} and α, or between P_{AR} and $p_{d_{tc_1}}$, can change slope depending on the values of λ_u, $p_{u_{t_c}}$, and T_r.

II. PARAMETER ESTIMATION FOR SYSTEMS SUBJECT TO PERIODIC CHECKOUT

A. Statement of the Problem

A system which undergoes a normal cycle of standby and checkout presents a number of theoretical and practical difficulties when the attempt is made to describe or predict the behavior, and in particular the availability, of the system. It is theoretically straightforward to specify an optimum period between checkouts, based on assumed failure rates during standby and checkout, but it is not so apparent how these rates are to be obtained. Whether failures "occur" during standby or checkout, if they are discovered only during checkout, they must be correctly assigned as being due to standby or checkout causes; otherwise, the maintenance plan may be ineffective or even worthless.

In addition to the problem of measuring failure rates, other equally important problems arise from factors which are known to have potentially strong effects on availability, and which must therefore be carefully considered in devising maintenance policy, but which are, unfortunately, difficult to measure. Errors occur during checkout in deciding whether a good system is really good or a bad system really bad, and further errors are committed in repair. This means a system can enter standby in a failed condition, and therefore be unavailable for the entire standby period.

The problem, then, becomes one of sorting out these contributory factors, assessing their possible influence through mathematical analysis, and measuring their values in field operations, so that maintenance policy can be guided accordingly. As it does not appear likely that detailed data from engineering analysis will be available in the near future, attention was directed to statistical methods of parameter estimation (Reference 3). In order to discuss the measurement methods in detail, the model parameters (previously defined in References 1, 2 and 3) are reviewed below.

B. Parameter Definitions

Figure 1 is a schematic diagram of the sequence of events occurring in a periodic checkout policy.

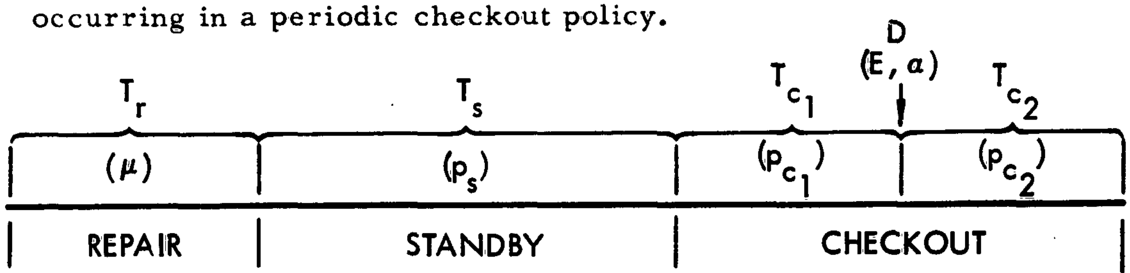

Figure 1. Time Sequence for a Periodically Checked System

The cycle is assumed to start with a repair period, followed by a standby period, after which there is a checkout period which is divided into two parts, corresponding to the time intervals before and after the point of "test decision," D, at which time the test unit is declared good or bad. The symbols in the figure are defined as follows:

T_r = Duration of repair period

T_s = Duration of standby period

T_{c_1} = Duration of checkout interval prior to test decision

T_{c_2} = Duration of checkout interval after test decision

D = Point (in time) of test decision

E = Probability that a unit which is failed at D will be declared bad at D

α = Probability that a unit which is good at D will be declared bad at D

μ = Probability that a unit is good at completion of repair

p_s = Probability that a unit which is good at entrance to standby is still good at completion of standby

p_{c_1} = Probability that a unit which is good at entrance to checkout is still good at D

p_{c_2} = Probability that a unit which is good at D is still good at completion of checkout

The interaction of these parameters can be seen by checking the different possible ways in which units can pass through the activities of Figure 1. For example, a unit can be repaired satisfactorily, fail during standby, but pass the checkout. If the actual condition of the unit during the time line sequence is described in terms of its condition at the points immediately after repair, immediately after standby, the point of test decision, and the end of checkout, there are five possible "histories" of the unit at these four points, where G means "good" and B means "bad": GGGG, GGGB, GGBB, GBBB, BBBB. No G can be preceded by a B, as there is no repair after the initial repair. It is assumed that the repair is of the replacement type, so that the condition of the (old) unit upon entering repair does not affect the probability that the (new) unit will be good upon exiting from repair.

With this many parameters, if the analysis is based only on the test decision results, the individual parameters cannot be estimated explicitly unless some variation occurs in the basic time line sequence of Figure 1. As noted in the Introduction, two specific variations were considered in Reference 3, and it was recommended in that document that the question of confidence limits for the parameters be investigated, using Monte Carlo procedures. These programs and the numerical results are described in the following two parts. A third method of parameter estimation is described in Reference 6. This method estimates parameters on the basis of the probability distribution of the number of cycles to first repair.

C. Variable Standby Time Method

This method of parameter estimation is applicable when a series of units experience different standby times, between checkouts. Given a minimum of three different standby times among a group of systems for which data are available, Reference 3 provides equations for estimating E, λ, and $\mu p_{c_1}(\alpha - E)$. The data required to make these estimates is, for each system, the standby time and whether or not the system passed the checkout.

If there are exactly three different standby times, whose lengths are in the ratio one-two-three, the parameter estimation equations

can be solved analytically. The solution of the equations is provided in Reference 3. Parameter estimation in this case is therefore straightforward, and the accuracy of estimation will depend only on the "noise" in the data due to its random nature. The average standby time should be of the order of $1/\lambda$; otherwise large inaccuracies are possible unless the sample is extremely large.

For more than three standby times, the equations whose solution is required for the maximum liklihood estimates cannot be solved analytically. A series of computer search routines was therefore developed by the Applied Mathematics Department, Programming and Applied Mathematics Laboratory (STL), as reported in Reference 7. The solution of these equations proved to be exceedingly difficult and time consuming, so that time did not allow more than a few preliminary computer runs for specific examples. These examples all used expected values as the input data, and in most cases, the right answers were obtained. These examples are discussed in Reference 7. A later example, in which "noise" was introduced into the data through a randomization process based on expected values, provided inconclusive results.

A program is available which is workable under most circumstances, should the opportunity arise to process data pertaining to variable standby times. For a further discussion of this program and its limitations, the reader is referred to Reference 7.

D. Multiple Checkout Method

The derivation and use of this method of parameter estimation require that three consecutive checkouts are performed following standby, as shown in Figure 2.

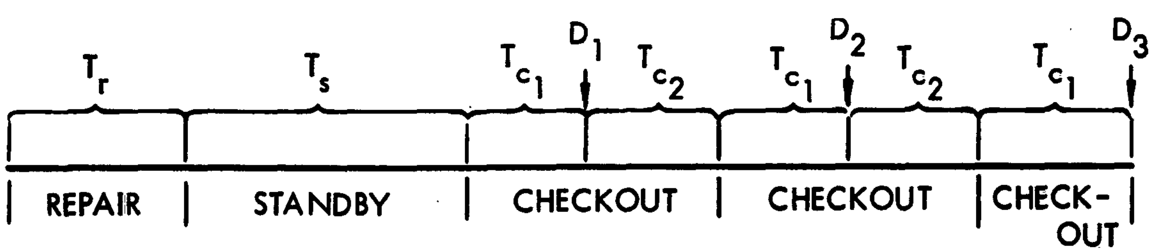

Figure 2. Time Sequence for a System Undergoing Multiple Checkouts

As before, the cycle is assumed to begin with a repair, followed by a standby period. This is followed by three consecutive checkouts, with no repair being performed, regardless of the test decisions at D_1, D_2, and D_3. Note that the actual condition of a unit can change (from good to bad) in passing from one checkout to the next, since failures can occur during checkout.

Three test decisions are thus obtained for each unit undergoing the sequence of Figure 2. These individual records can then be grouped into eight categories, corresponding to the eight possible outcomes at D_1, D_2, and D_3, where P denotes passing the test and F denotes failing: (PPP), (PPF), (PFP), (PFF), (FPP), (FPF), (FFP), and (FFF).

The probabilities of these sequences can be written in terms of the variables shown in Figure 2. For example, by considering the mutually exclusive ways of producing the sequence (PFP), the probability is obtained as

$$P_r(PFP) = \mu p_s p_{c_1}^3 p_{c_2}^2 \alpha(1-\alpha) + \mu p_s p_{c_1}\left(1 - p_{c_1}p_{c_2}\right)(1-\alpha)(1-E)$$

$$+ \mu p_s p_{c_1}^2 p_{c_2}\left(1 - p_{c_1}p_{c_2}\right)\alpha(1-\alpha)(1-E) + \left(1 - \mu p_s p_{c_1}\right)(1-E)^2 E$$

For convenience, as in Reference 3, the variables $x = \mu p_s p_{c_1}(\alpha - E)$ and $t = p_{c_1}p_{c_2}$ will be introduced. Equations (1) to (8) below give the probabilities of the sequences in terms of E, α, x and t.

$$P_r(PPP) = -(1-\alpha)xt\left[t(1-\alpha) + (1-E)\right] + (1-E)^2(1-E-x) \qquad (1)$$

$$P_r(PPF) = (1-\alpha)\,xt\left[t(1-\alpha) - E\right] + E(1-E)(1-E-x) \qquad (2)$$

$$P_r(PFP) = -(1-\alpha)\,xt\left[\alpha t - (1-E)\right] + E(1-E)(1-E-x) \qquad (3)$$

$$P_r(PFF) = (1-\alpha)\,xt(\alpha t + E) + E^2(1-E-x) \qquad (4)$$

$$P_r(FPP) = -\alpha xt\left[t(1-\alpha) + (1-E)\right] + (1-E)^2(E+x) \qquad (5)$$

$$P_r(FPF) = \alpha x t\left[(1-\alpha)t - E\right] + E(1-E)(E+x) \qquad (6)$$

$$P_r(FFP) = -\alpha x t\left[\alpha t - (1-E)\right] + E(1-E)(E+x) \qquad (7)$$

$$P_r(FFF) = \alpha x t(\alpha t + E) + E^2(E+x) \qquad (8)$$

In these equations, it is of interest to note the duality property, that if all P's are changed to F's for any given expression, and vice versa, the correct new expression is obtained by merely substituting $1-\alpha$ for α and $1-E$ for E (and, therefore, $-x$ for x, since x involves α and E through the term $\alpha - E$). For example, by comparing the equations for $P_r(PPP)$ and $P_r(FFF)$, it is seen that the latter is obtained from the former by substituting $1-\alpha$ for α, $1-E$ for E, and $-x$ for x.

1. Parameter Estimation Equations

The above equations can be solved to obtain estimates for E, α, x ant t. The estimates used in the computer analysis to be described are as follows:

$$\hat{E} = \frac{(PFF) - (FPF)}{(PFF) - (FPF) + (PFP) - (FPP)} \qquad (9)$$

$$\hat{\alpha} = \frac{(FPF) - (FFP)}{(FPF) - (FFP) + (PPF) - (PFP)} \qquad (10)$$

$$\hat{t} = \frac{(FPF) - (FFP) + (PPF) - (PFP)}{(PFF) - (FPF) + (PFP) - (FPP)} \qquad (11)$$

$$\hat{x} = \frac{(FPP) + (FPF) + (FFP) + (FFF)}{M} - \hat{E} \qquad (12)$$

In these equations, the symbols (PFF), (FPF), etc., represent the numbers of units experiencing those particular sequences, and M denotes the total number of units in the sample (M = (PFF) + (FPF) + · · ·).

As pointed out in Reference 3, of the parameters E, α, μ, p_s, p_{c_1}, and p_{c_2}, only E and α are obtained directly from the Multiple Checkout Method, unless some of the standby times are 0, or unless some knowledge is assumed about one or more of the parameters. Estimates of certain combinations of the other parameters are also obtained.

Attention was focused on the parameters E and α in setting up the computer runs, and the following discussion reflects this fact. However, it should be emphasized that the other parameters can be estimated under certain conditions, and the ability to measure these parameters can be investigated through a modification to the existing computer program. First, assume that immediately after repair, all units are subjected to three consecutive checkouts. All parameters except p_s can then be estimated if some relationship is known between p_{c_1} and p_{c_2} (or if the value of one of these parameters is known). For example, it might be known that $p_{c_1} = p_{c_2}^2$.

Or, if the test decision occurs at or near the end of checkout, or if the environmental stress is minimal in checkout after the point of test decision, it might be assumed that $p_{c_2} \approx 1$. As an estimate for $t = p_{c_1} p_{c_2}$ is available, we can then estimate both p_{c_1} and p_{c_2}. Knowing α, E, and p_{c_1}, the estimate of $x = \mu p_{c_1}(\alpha - E)$ can be used to estimate μ. This completes the list of unknown parameters, except for p_s.

Second, if a group of similar units pass from repair into standby for some fixed period, and then are given a single checkout, the data from these units can be combined with that from the units above, which had three checkouts without entering standby, to estimate p_s. The estimate of $x = \mu p_s p_{c_1}(\alpha - E)$ for the group undergoing standby when divided by the estimate of $x = \mu p_{c_1}(\alpha - E)$ for the group without standby gives an estimate of p_s.

When estimates are available for all of these parameters, an estimate can also be made for availability. Thus, the accuracy of estimating availability could be analyzed. However, as it was not an objective of the present computer program to investigate these possibilities, only a few hand calculations were applied to the output from the present program. (See Paragraph 3(d) below.)

2. Computer Program

A computer program was devised to simulate test results in order to check the characteristics of the estimating Equations (9) to (12). In the program, a specified number of units is processed

through the activities shown in Figure 2, and for each unit a random number is generated at each critical point in the time sequence to determine what actually happens. For example, for the first unit a random number is drawn and compared with μ (the probability of successful repair). If the number is smaller than μ, the repair is successful and another random number is drawn and compared with p_s (the probability of surviving standby), and so on. If the original random number had not been less than μ, the repair would have been unsuccessful. Since the unit was then in a failed condition, it would remain in a failed condition for the rest of the sequence of events, as no repair is performed. Therefore, no further random numbers would be drawn until the first test decision, where a number would be drawn and compared with E. Numbers would similarly be drawn at the next two points of test decision. Thus, the minimum number of random numbers generated in the sequence is four—one at repair and one at each test decision—whereas, if the unit survives the entire sequence (except, perhaps, for the last check period), ten random numbers will be required.

A series of individual histories is generated in this manner. While the computer "knows" which particular units are good and which bad, this information is not printed out. As in an actual set of test data, the data printed out by the program is the number of units passing and failing at each test decision, and the numbers of units with each of the eight possible test histories, (PPP), (PPF), etc.

The data output from this program, written for the 7090 computer, is extremely rapid. About 90, 000 items can be run through the test sequence in one minute of machine time. This includes the calculation of means and variances of the estimators and the printout of results. Because of the computation speed, it was possible to investigate a large number of cases with different parameter values, and a satisfactory range of sample sizes.

The program input requires specification of the parameters α, E, μ, p_s, p_{c_1}, and p_{c_2}; sample size, M; and the number of runs, N, to be made with all of the above numbers held fixed. The printed output is in two parts, a summary printout for N runs, and an optional detailed printout for each run. The summary printout lists as a heading the true values of the input parameters and sample size, and the true (computed) values of $t = p_{c_1} p_{c_2}$ and $x = \mu p_s p_{c_1} (\alpha - E)$. The following statistics are then printed out for each of the estimators, $\hat{\alpha}$, $\hat{E}$, $\hat{x}$, and $\hat{t}$: average (for the M units and N runs), variance from average, variance from true, maximum for each parameter, minimum for each parameter, and the fraction of values within 5, 10, 20 and 50 percent of the average. In addition, the average, variance, maximum, and minimum of the numbers of units failing each of the three checkouts is printed. The detailed printout lists for each run the number of units in each possible sequence PPP, PPF, etc., and the values of each of the estimators.

Note that the program does not print out the numbers of units actually good and bad, but only the numbers of units passing and failing each checkout.

3. Discussion of Results

Confidence limits cannot be prescribed for any of the parameters unless there is the possibility of real failures during checkout. This is not a serious limitation, as generally this possibility will exist. The Multiple Checkout Method is most useful when there is a significant change in the state probabilities from checkout to checkout. This is because of the nature of the estimating equations, which degenerate when there are no failures during checkout. If $p_{c_1} = p_{c_2} = 1 (t = 1)$, Equations (1) to (8) reduce to four equations, since the probability of a particular sequence of P's and F's does not depend on their order; therefore, $P_r(PPF) = P_r(PFP) = P_r(FPP)$ and $P_r(PFF) = P_r(FPF) = P_r(FFP)$. With these probabilities equal, the estimating Equations (9) to (11) have an expectancy of 0 in both the numerator and the denominator. Also, Equation (12) depends on (9). It will therefore be assumed that $t \neq 1$.

The numerical results are tabulated in Appendix A. Not all of the computer summary output is shown because of space requirements. The tables show the mean and variance from average for $\hat{E}$ and $\hat{\alpha}$ for each case. The true values of the parameters used in each case are shown, and the values for M (number of units in the sample) and N (number of runs). The number of runs required to provide an approximately correct value for the variances was determined by a preliminary series of runs in which M was held fixed and N was varied to indicate at what point the variance does not differ widely from one series of N runs to the next. The larger the true variance is, the larger N should be to estimate it accurately; but as it is of no value to have accurate estimates of large variances, a compromise was made in which N is large enough to approximate the true variance, providing it is small enough to be of some use. Therefore, in the tables, variances larger than about 0.1 may be inaccurate.

a. Combined Variation of p_s, α, and E (Tables A-1 to A-9)

The first series of tables, Tables A-1 to A-9, report the results of the most extensive series of runs, which were for combinations of the following parameter values:

$$E = 1.0,\ 0.9,\ 0.7$$

$$\alpha = 0,\ 0.1,\ 0.3$$

$$p_s = 1.0,\ 0.8,\ 0.5$$

$$p_{c_1} = 0.8,\ p_{c_2} = 1.0,\ \mu = 1.0$$

For the combination $\alpha = 0$, $E = 1$, the estimates are exact: $\hat{\alpha}$ becomes indentically 0 and $\hat{E}$ identically 1. Therefore, this combination was not run. Since μp_s always appears as a product in the equations, the tables are the same as if p_s were held fixed and μ varied.

The tables show the marked effect of the true value of E on the accuracy of estimating both E and α, and, similarly, for the

true value of α. For $p_s = 1$, if $E = 1$ and $\alpha = 0.1$, a reasonably accurate estimate of both E and α is obtained with $M = 300$; whereas if $E = 0.7$, the sample size must be increased to about 1000 for comparable accuracy—similarly, for a change in α from 0 to 0.3, with $E = 0.9$. This is shown in Table 1 below.

Table 1. Effect of True Values of E and α on Estimation Accuracy

E	α	M	$\hat{E}$ Variance	$\hat{\alpha}$ Variance
1.0	0.1	100	0.1562	0.0242
		300	0.0149	0.0053
		500	0.0081	0.0032
		1000	0.0035	0.0015
0.7	0.1	100	0.5742	0.6026
		300	0.0625	0.4254
		500	0.0269	0.1992
		1000	0.0118	0.0163
0.9	0	100	0.0096	0.1023
		300	0.0025	0.0097
		500	0.0018	0.0063
		1000	0.0009	0.0031
0.9	0.3	100	1.2113	0.9668
		300	0.7569	0.2929
		500	0.0444	0.0575
		1000	0.0148	0.0152

The table shows that if only one of the two types of error is present—even though this fact is not known—the estimate for the other type of error becomes much more accurate. If it is known that only one type of error is present, Equations (1) to (8) become different, and give rise to different estimators for E or α (whichever is present). If it is known that $\alpha = 0$, an estimator for E is

$$\hat{E} = \frac{(FFF)}{(FFF) + (FPF)} \tag{13}$$

This estimator has a smaller variance than that given by Equation (9); for one thing, it is seen that Equation (13) must lie between 0 and 1, whereas Equation (9) frequently gets bigger than 1.

If it is known that E = 1, an estimator for α is

$$\hat{\alpha} = \frac{(PFP)}{(PFP) + (PPP)} \tag{14}$$

This is a better estimator than Equation (10), and lies between 0 and 1, whereas Equation (10) frequently becomes negative. The properties of these estimators were not investigated on the computer.

As p_s decreases from 1, other parameters being held constant, the tables show how the accuracy of estimating E and α diminishes. Table 2 was constructed to indicate this fact as well as to show the combined effects of varying E, α, and p_s. In Table 2, M is fixed at 500, $\mu = 1$, $p_{c_1} = 0.8$, and $p_{c_2} = 1$. The table demonstrates a large effect on accuracy as E, p_s, and $1 - \alpha$ decrease from their maximum values. When $p_s = 1$, $\hat{\sigma}_\alpha^2 = 0.0104$ when $\alpha = 0.1$ and E = 0.9, and when $p_s = 0.5$ for the same values of α and E, $\hat{\sigma}_\alpha^2 = 0.4287$. Or, when $p_s = 0.8$, if $\alpha = 0$, as E decreases to 0.7, $\hat{\sigma}_E^2$ increases to only 0.0134, and if E = 1, as α increases to 0.3, $\hat{\sigma}_E^2$ increases to 0.0511; but if both effects occur simultaneously, σ_E^2 increases to 1.3742. As mentioned earlier, values of large variances are not accurate. This is indicated here, since this value (1.3742) is less than the corresponding value for $p_s = 1$, when it can be expected to be larger. Another anomaly is seen for the value of $\hat{\sigma}_\alpha^2$ when $p_s = 0.5$, E = 0.9 and $\alpha = 0.3$, which value is less than the corresponding value for $p_s = 0.8$. An even more obvious case is the decrease in $\hat{\sigma}_\alpha^2$ for increasing α when $p_s = 0.5$ and E = 0.7.

Table 2. Effects of Combined Parameter Variations on Estimation Accuracy

($M = 500$, $\mu = p_{c_2} = 1.0$, $p_{c_1} = 0.8$)

$p_s = 1.0$

$\hat{\sigma}_E^2$

E \ α	0	0.1	0.3
1.0	--	0.0081	0.0353
0.9	0.0018	0.0107	0.0444
0.7	0.0067	0.0269	1.7154

$\hat{\sigma}_\alpha^2$

E \ α	0	0.1	0.3
1.0	--	0.0032	0.0192
0.9	0.0063	0.0104	0.0575
0.7	0.0195	0.1992	0.7782

$p_s = 0.8$

$\hat{\sigma}_E^2$

E \ α	0	0.1	0.3
1.0	--	0.0113	0.0511
0.9	0.0027	0.0150	0.1718
0.7	0.0134	0.1027	1.3742

$\hat{\sigma}_\alpha^2$

E \ α	0	0.1	0.3
1.0	--	0.0039	0.0321
0.9	0.0203	0.0268	0.7532
0.7	0.2489	0.4182	1.4979

$p_s = 0.5$

$\hat{\sigma}_E^2$

E \ α	0	0.1	0.3
1.0	--	0.0181	0.3281
0.9	0.0073	0.0555	1.6381
0.7	0.6273	0.8417	2.9079

$\hat{\sigma}_\alpha^2$

E \ α	0	0.1	0.3
1.0	--	0.0072	0.0579
0.9	0.1554	0.4287	0.6936
0.7	3.6239	2.5981	2.0662

b. Effects of Other Parameters (Tables A-10 to A-13)

As regards the effects of the other parameters, μ, p_{c_1}, and p_{c_2}, it was mentioned earlier that μ has the same effect as p_s, and that if $p_{c_1} = p_{c_2} = 1$, no estimates can be made. The values chosen in the tables just discussed, Tables A-1 to A-9, were $p_{c_1} = 0.8$ and $p_{c_2} = 1.0$. These values were not chosen on the basis of providing good estimates for E

and α, as other values are known which will give better estimates. They were meant to represent "reasonable" values for the common situation in which the test decision occurs near the end of checkout, or, in general, where there is little chance of failure after test decision. For a fixed value of $t = p_{c_1} p_{c_2}$, the lower p_{c_2} is, the more accurate the estimate of either $\hat{E}$ or α. This unexpected result can be interpreted to mean (assuming equal environmental stresses before and after test decision) that the earlier in time that the test decision is made, the better. This is not to say, however, that "snap judgment" is preferred, as it is assumed that the true E and α are not a function of how quickly the decision is made.

Tables A-10 and A-11 show the results for $p_{c_1} = 0.5$, $p_{c_2} = 1.0$, and $p_{c_1} = 1.0$, $p_{c_2} = 0.5$, respectively, with the parameters E, μ, and p_s at their maximum values, and for various values of α. It is seen that good estimates of α are obtained with $p_{c_1} = 1.0$, $p_{c_2} = 0.5$ (Table A-11), even for a sample size of 100. Worse values are obtained for α and E when the values for p_{c_1} and p_{c_2} are reversed (Table A-10). Table A-12 shows that increasingly better estimates of E are obtained as p_{c_2} decreases, the best case being when $p_{c_2} = 0$. Estimates for this case are listed in Table A-13. A sample of 50 or even 25 is adequate, as shown in the table. (Although M = 50 is the lowest value shown in the table, a comparison of the variance with M shows a linear relationship which can be used to extrapolate or interpolate to other values of M.) In Tables A-12 and A-13, $\hat{\alpha}$ is identically 0 and therefore is not shown.

c. <u>Comparison of Favorable Cases for α and E (Tables A-14 and A-15)</u>

Tables A-14 and A-15 show the effect of varying the other parameters for the "favorable cases" (i.e., parameter combinations allowing good estimates) for E and α. In these tables, as in other tables in the appendix, parameter values

are not repeated from case to case if they do not change. For example, in Table A-14, the second line specifies $E = 0.9$. This means all other parameters (including M and N) remain the same. Study of these tables shows that for fixed values of the other parameters, $\hat{\sigma}_\alpha^2$ increases approximately linearly with α, and $\hat{\sigma}_E^2$ increases approximately linearly with $1 - E$; and both decrease approximately linearly with increasing sample size. For example, for $E = \mu = 1$, $\hat{\sigma}_\alpha^2 \approx 5\alpha/M$. This relationship could be used to find the approximate sample size required to measure α with a specified accuracy. If a simple criterion is used, such as $\hat{\sigma}_\alpha^2 \sim \alpha/2$, then it is found by combining these two equations that $M \approx 20/\alpha$. This shows that even though (as found previously) the accuracy of measuring α increases with decreasing α, the relative accuracy (in terms of fractional error of the true value) decreases. That is, using the relationship between M and α, we have

$$\alpha = 0.05, \ M = 400$$

$$\alpha = 0.1, \ M = 200$$

$$\alpha = 0.2, \ M = 100$$

all giving the same relative accuracy of measuring α, namely, a standard deviation of 1/2 the true value.

A comparison of the tables for E and α shows that, other things being equal, it is easier to measure E accurately than α. This is for two reasons: (1) $\hat{\sigma}_E^2$ is smaller than $\hat{\sigma}_\alpha^2$ under "similarly favorable" conditions, and (2) $E > \alpha$ for any useful system, so that the relative accuracy is greater for E even if the σ's were the same.

d. Estimate of Availability

Two samples of 20 runs were made under the condition that $p_{c_2} = 1$ and with two values of p_s, $p_s = 1$, and $p_s = 0.7$. Under these conditions, all parameter values are either known or estimated, so that an estimate can be made for system

availability (for an assumed checkout and repair time). Table A-16 shows the twenty sets of estimates obtained for E, α, p_{c_1}, μ, p_s and λ_s (for an assumed standby time of 1 unit, i.e., $\lambda = -\ln p_s$). The estimates for all parameters except p_s were obtained from the set of runs with $p_s = 1$, as this is a more favorable case. The estimates of p_s were obtain by matching pairs in order of occurrence in the two sequences of runs, but any order could have been used.

The estimates for availability shown in the table were obtained from the equation (Reference 5, p. 7):

$$A = \frac{P(G)\left(\dfrac{1 - e^{-\lambda_s T_s}}{\lambda_s}\right)}{T_s + T_c + T_r\left[E + P(G)e^{-\lambda_s T_s} p_c(\alpha - E)\right]}$$

where

$$P(G) = \frac{E\mu}{1 - e^{-\lambda_s T_s} p_c\left[1 - \alpha + \mu(\alpha - E)\right]}$$

Since $p_{c_2} = 1$ and $p_c = p_{c_1}$, and estimates of all other parameters are provided in Table A-16, system availability can be estimated for assumed values of T_s, T_r, and T_c, as shown in the right-hand column of Table A-16. The sample size for this table is M = 1000.

e. <u>Normal Approximation to Distribution of Estimators</u>

To obtain confidence limits, the normal distribution can be applied to the estimators if the variance is of the order of 0.01 or less (i.e., standard deviation = 0.1 or less). An example of applying this approximation is given in Table 3 below, for the case M = 300, E = 1.0, $\mu = 1.0$, $p_s = 1.0$, $p_{c_1} = 0.8$, $p_{c_2} = 1.0$, $\alpha = 0.1$.

Table 3. Fraction of Values Within P Percent of True Value

	$\hat{E}$		$\hat{a}$	
P Percent	Actual	Normal Approx	Actual	Normal Approx
5	0.326	0.326	0.048	0.040
10	0.608	0.600	0.108	0.112
20	0.914	0.905	0.224	0.221
50	1.000	1.000	0.554	0.516

f. Estimator Bias

In calculating the values of the estimators and their variances on the computer runs, no truncation was performed on estimates greater than one or less than zero. Since the parameters are probabilities, in any actual case where the estimate was greater than one or negative, it would be rounded to one or zero, respectively. This could reduce the variance about the true parameter value, but would tend to bias the estimates so that the expected value would not equal the true value. The reason for this is that the present untruncated estimates are apparently unbiased. The variances shown in the tables are about the average, and these would not necessarily decrease under truncation. This was not investigated on the computer. Without truncation, it was found that there was little difference between the variances about the average and about the true values.

III. PARAMETRIC ANALYSIS OF AVAILABILITY OF A PERIODICALLY CHECKED SYSTEM

A. Introduction

In this section the variables which affect the availability of a periodically monitored system will be discussed on the basis of a computer analysis which was performed in which the parameters were allowed to vary individually. To aid in parameter definition, Figure 3 illustrates again the sequence of events assumed for a system subject to periodic checkout. This figure is similar to Figure 1, but additional parameters are indicated.

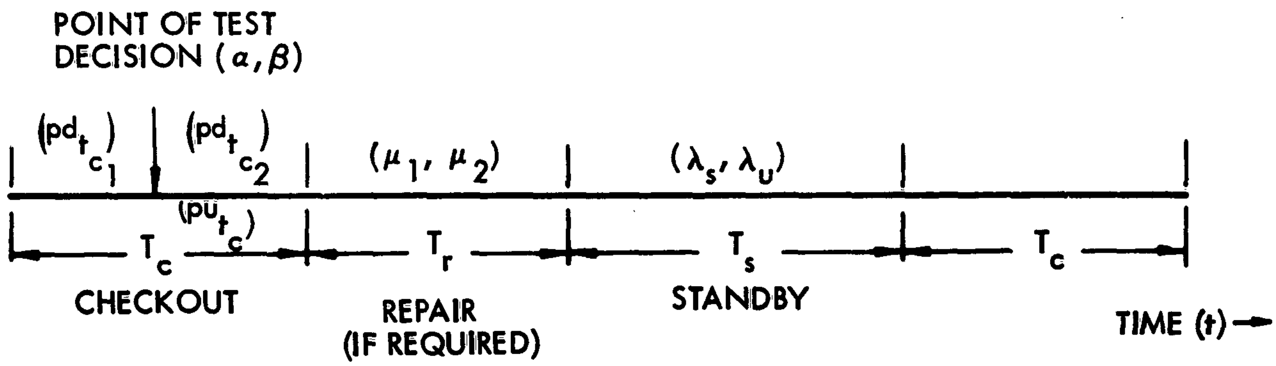

Figure 3. Time Sequence for a Periodically Checked System

The symbols appearing in this diagram are defined as follows:

P_{AR} = probability of alert readiness, or probability the system is in standby and nonfailed at a random point in time

T_s = duration of standby period

T_c = duration of checkout period

T_r = duration of repair period

α = probability of calling a system with no detectable failures bad during checkout

β = probability of calling a system with a detectable failure good during checkout*

λ_s = rate of occurrence of detectable failures during standby

*The parameter β used in this section equals 1 - E of the previous section.

λ_u = rate of occurrence of undetectable failures during standby

μ_1 = probability a repaired system is unfailed

μ_2 = probability a repaired system is failed detectably

$1-\mu_1-\mu_2$ = probability a repaired system is failed only undetectably

$p_{d_{t_{c_1}}}$ = probability of no detectable failures occurring during first portion of checkout (prior to test decision)

$p_{d_{t_{c_2}}}$ = probability of no detectable failures occurring during second portion of checkout (after test decision)

$p_{d_{t_c}}$ = probability of no detectable failures occurring during checkout = $p_{d_{t_{c_1}}} p_{d_{t_{c_2}}}$

$p_{u_{t_c}}$ $\doteq$ probability of no undetectable failures occurring during checkout

$p_{d_{t_s}}$ = probability of no detectable failures occurring during standby = $\exp-(\lambda_s T_s)$

$p_{u_{t_s}}$ = probability of no undetectable failures occurring during standby = $\exp-(\lambda_u T_s)$

As in Part II of this report, the list includes parameters representing the possibility of errors during checkout, failures induced by checkout, imperfect repair, and failures during standby. In addition, the parameters λ_u, $1 - \mu_1 - \mu_2$, and $p_{u_{t_c}}$ are introduced to allow for the possibility of failures during standby, repair and checkout which are inherently undetectable by the checkout procedures employed to monitor the system. This characteristic is called "partial test coverage," and is different from failures undetected due to errors in equipment or personnel, as accounted for by the parameter β. In the model, a system failure of the type being represented by λ_u is never detected by the checkout procedures employed, and will only be discovered later (if at all) at a rear echelon when more thorough tests are performed. Though not detected, failures of this type may be corrected because of the occurrence of other (detectable) failures, or a false alarm, both of which lead to repair (or replacement). For those modes of failure which are presumably covered by the checkout, some will occasionally be missed due to error, and this is specified to occur with a probability β.

As seen in the list of definitions, the introduction of undetectable failures changes the meaning of α (and β) slightly from that of Part II; i.e., it is no longer the "probability of calling a good system bad," since some of the "good" systems which are called bad through "error," are actually bad because of undetectable failures; thus, partial test coverage together with false alarms can lead to correct decisions. This slight change in the meaning of α will be shown later to be very significant.

Referring to Figure 3, the system is assumed to be assigned to a fixed alert (or standby) period T_s, during which it is not monitored. It is then checked out, which requires a fixed time T_c. This time is divided into two parts, corresponding to the times before and after test decision. If the decision is made that the system is bad, it enters repair; otherwise, it re-enters standby. Upon completion of repair, which requires a fixed time, the system re-enters standby without a subsequent checkout. The repair period may include some checkout activities, but these are not an explicit part of the model. If there were a specific provision for checkout after repair within the model, then the repair period would have to be a variable, instead of fixed as assumed here, since successive checkout and repair activities could occur an arbitrary number of times within the repair period itself.

The system is assumed to fail exponentially during standby, for both detectable and undetectable failures.

Since a fixed standby period is assumed, the system is not operating under a "calendar" maintenance policy, i.e., it is not possible, in general, to predict the future time intervals to which the system will be assigned to standby. It is also noted, from the definition of P_{AR} in the list above, that when a system is in checkout, even though it is good, it is not considered available for its mission.

The alert readiness of a system subject to the above conditions is given by Eq. (54), p. 135 of Reference 9, which is derived in Reference 1. This equation is shown in Figure 4. The complexity of the equation made it desirable to develop a computer program to analyze

numerically the effects of the numerous parameters on availability and their interactions. The remainder of this section describes the computer program and discusses the results of the numerical analysis.

$$P_{AR} = \cfrac{\cfrac{\mu_1 p_{u_{t_c}} p_{d_{t_c}} (1-\beta)\left[1-(1-\alpha)p_{d_{t_s}} p_{d_{t_c}}\right]\left[1-e^{-(\lambda_s+\lambda_u)T_s}\right]}{\left[1-\bar{p}_{u_{t_s}}\bar{p}_{d_{t_s}}\bar{p}_{u_{t_c}} p_{d_{t_c}}(1-\alpha)\right]\left[1+\bar{p}_{d_{t_s}} p_{d_{t_{c_1}}}(1-\mu_2)(1-\beta-\alpha)-p_{d_{t_s}} p_{d_{t_c}}(1-\alpha)\right](\lambda_s+\lambda_u)}}{T_s+T_c+\cfrac{(1-\beta)\left[1-(1-\alpha)\,p_{d_{t_s}}\bar{p}_{d_{t_c}}\right]T_r}{1+\bar{p}_{d_{t_s}}\bar{p}_{d_{t_{c_1}}}(1-\mu_2)(1-\beta-\alpha)-\bar{p}_{d_{t_s}}\bar{p}_{d_{t_c}}(1-\alpha)}}$$

Figure 4. Alert Readiness of Periodically Checked System

B. Computer Program

To aid in the interpretation of results, the program was written to allow automatic plotting of the computer output in graphical form. Some of these graphs are reproduced in Appendix B and will be discussed shortly. All graphs plot P_{AR}, probability of alert readiness, as the ordinate, and one of the twelve independent variables as abscissa. One of the other independent variables is allowed to vary, with all others held fixed, and can assume up to seven different values for any one chart. Each figure in Appendix B, therefore, has at most seven curves, each curve representing the relationship between P_{AR} and one independent variable for a particualr value of some other independent variable (the parameter). The parameter changes from curve to curve, the values being indicated at the top of each graph. Each value is associated with a symbol used in plotting the curve. The values are written in terms of a three-digit figure followed by a two-digit figure, the latter representing a power of ten. For example, 0. 300 00 is 0. 3, 0. 150 02 is 15, and 0. 100 -02 is 0. 001.

The values of the other parameters, which are held fixed for each graph, are also indicated on the graphs.

C. Discussion of Results

1. Variable T_s, Parameter α (Figures B-1 to B-13)

When P_{AR} is plotted against T_s, there will always be some T_s which maximizes P_{AR} (including the cases where the maximum

occurs at $T_s = \infty$). Basically, this value of T_s represents an optimum trade-off between time lost from readiness due to checkout (and repair of apparent failures induced by checkout) and time lost from readiness due to failure in standby. However, there are other, more complex, relationships in the trade-off picture: when errors are committed during checkout in deciding whether the system is good or bad, when failures are induced during checkout which go undetected, and when repair is faulty so that a repaired system can enter standby in a failed condition. Some of these complex interactions are illustrated when the parameter α varies.

Figures B-1 through B-5 show P_{AR} versus T_s for different values of α, each figure being for different values of T_c and T_r. Figure B-1 is for $T_c = 10$, $T_r = 1$, implying that checkout is a relatively time-consuming process and that when a failure is found, it is repaired quickly (as might be characteristic of replacement type repair). Since repair is so rapid, it might be expected that α would have little effect, since no significant time is lost by calling a good system bad. This lack of sensitivity is clear from the figure. Although there is slight variation with α, it is noteworthy that the highest value of α (i. e., the highest probability of calling a good system bad) gives the highest value of P_{AR}. This will be discussed further below. In fact, later results will show that the above remarks need to be qualified.

Figures B-2 to B-5, which have increasing ratios of T_r to T_c, illustrate increasingly greater effects of α. Figure B-4 is for the same values of T_c and T_r as Figure B-3, but $\beta = 0.5$ instead of 0.1. Figure B-5, in addition to having a larger value of T_r, has a larger value of λ_u. These figures also illustrate a "crossover" phenomenon: for small T_s, P_{AR} tends to increase as α gets smaller, and for large T_s, this relation is reversed. This can be explained—as can the fact that the highest α gave the highest P_{AR} in Figure B-1—as being due to the presence of undetected failures in the system (the parameters λ_u, $p_{u_{t_c}}$, and $1 - \mu_1 - \mu_2$). Although α is called a "false alarm" probability, it

can also be considered as a type of preventive maintenance factor. This has been previously pointed out in Reference 8. Usually, preventive maintenance pertains to the elimination of incipient failures, whereas α eliminates possibly existing but undetectable failures; otherwise, the similarity is apparent. For example, an optimum replacement policy (a form of preventive maintenance) can be determined by setting $\alpha = 1$ (and $\beta = 0$).

With this interpretation of α, Figures B-1 to B-5 are more understandable. For sufficiently large standby periods, $\alpha \rightarrow 1$ (for maximum P_{AR}) if there are any undetectable failures possible during standby; and for any T_s, if T_r is small and/or undetectable failures are likely, replacement may be optimum. (Of course, we are not considering here the added burden on the logistics system of returning a lot of good units for repair.)

In Figure B-5, note that as α increases, the optimum T_s first decreases, then increases. This effect is shown more clearly in Figure B-6, for which $T_r = 5$ instead of 25. The change in slope of P_{AR} with α as T_s increases is shown in Figures B-7, B-8, and B-9, whose curves represent cross sections of Figures B-3, B-4, and B-5, respectively, at fixed values of T_s.

While it is clear from the figures (particularly Figures B-5 and B-6) and the above remarks that undetectable failures during standby make it desirable to replace the system periodically, the question arises as to the effect of α when there are undetectable failures during checkout and repair. The effect of undetectable failures during checkout ($p_{u_{t_c}}$) is shown by a comparison of Figure B-10, where $p_{u_{t_c}} = 1$ (and there are no other types of undetectable failures present either) and Figure B-11, where $p_{u_{t_c}} = 0.6$. When no undetectable failures of any type are present, as in Figure B-10, P_{AR} achieves its maximum when $\alpha = 0$, as expected. If $p_{u_{t_c}}$ is reduced to 0.6, the main effect is to greatly increase the optimum time between checkouts, in order to reduce the frequency of introducing undetectable failures.

(Of course, when $\alpha = 1$, P_{AR} becomes independent of $p_{u_{t_c}}$.) When a checkout does occur, however, the system should automatically go into repair ($\alpha = 1$). Obviously, there is a conflict between the criteria for optimum checkout time for $p_{u_{t_c}}$ as related to λ_u.

Figures B-12 and B-13 show the effect of α when undetectable failures occur during repair. In Figure B-12, $\mu_1 = 0.6$ and $\mu_2 = 0.4$, so that no undetectable failures occur during repair. In Figure B-13, μ_2 is reduced to 0.2, for the same μ_1, which implies that half the failures induced by repair are undetectable. Comparison of the figures shows that this just has the effect of reducing P_{AR}, without strongly affecting the optimum checkout period and without making periodic replacement desirable ($\alpha = 0$ is best). The reason for this is clearly that undetectable failures are introduced only through replacement/repair.

2. Variable T_s, Parameters β, μ_1, and λ_s (Figures B-14 to B-20)

For a fixed value of T_s, P_{AR} is monotonically decreasing with increasing β, λ_s, and λ_u, and with decreasing μ_1, as shown in Figures B-14 to B-20. The optimum checkout period decreases substantially as β, λ_s, or λ_u increases, whereas it does not change with μ_1; i.e., more frequent checkout does not help if repair becomes less (or more) reliable. The coincidence of the P_{AR} peaks for varying μ_1 is seen more clearly in Figure B-17, where $T_c = 10$ and $T_r = 1$.

Figures B-18, 19, and 20 are cross sections of Figures B-14, B-15, and B-16, respectively. Figure B-18 shows that β has much less effect for small T_s than for large T_s, since if a bad item is not detected, it probably will be at the next checkout, and if there is not much time between checkouts, there is a smaller reduction in readiness. Figure B-19 illustrates the almost linear relationship between alert readiness and repair effectiveness and shows that the slope changes slowly with increasing T_s. Figure B-20 describes the known fact that standby failure rate has an increasingly large effect as time between checkouts increases.

3. Variable T_s, Parameters λ_u, p_{ut_c}, and $1-\mu_1-\mu_2$ (Figures B-21 to B-25)

The effects of changing undetectable failure rates are shown in Figures B-21 to B-25. The change in P_{AR} for different λ_u's and $\alpha = 0.1$ is shown in Figure B-21. The values of the lower curves could be increased significantly, however, as discussed earlier, by increasing α. It was also previously remarked that decreasing p_{ut_c} increases the optimum period between checkouts; this is illustrated in Figure B-22. The probability of undetectable failures during repair, $1 - \mu_1 - \mu_2$, is varied in Figure B-23 by keeping μ_1 fixed at 0.6 and varying μ_2 between 0 and 0.4. As was the case with μ_1, there is no marked effect on optimum checkout period.

Figures B-25 and B-25 are cross sections of Figures B-22 and B-23, respectively (a cross section of Figure B-21 is not shown, as it does not differ largely from Figure B-20 for λ_s). As with μ_1, an approximately linear relationship holds between $1 - \mu_1 - \mu_2$ and P_{AR} for a given value of T_s.

4. Variable T_s, Parameters $p_{dt_{c1}}$ and $p_{dt_{c2}}$ (Figures B-26to B-31)

These parameters produce effects similar to some of those already discussed. $1-p_{dt_{c1}}$ is similar to α, and all of the remarks concerning α apply also to $1-p_{dt_{c1}}$. If $p_{dt_{c1}}$ is small, this means that good systems will tend to fail during checkout and go into repair, thus removing undetectable failures. Figure B-26 graphs P_{AR} versus T_s for different values of $p_{dt_{c1}}$. Note that for $2.5 \leq T_s \leq 7.5$, the bottom two curves represent the extreme values of $p_{dt_{c1}}$; the lowest curve is for $p_{dt_{c1}} = 1$, and the next higher curve is for $p_{dt_{c1}} = 0$. This implies that for a fixed T_s in this region, there is an optimum value of $p_{dt_{c1}}$ other than 0 or 1. This effect is shown in Figure B-27, which is a cross section of Figure B-26 for $T_s = 5$. In Figure B-27, P_{AR} is shown as a function of $p_{dt_{c1}}$ for $T_s = 5$, for different values of λ_u. It is seen that as λ_u increases, the optimum $p_{dt_{c1}}$ decreases, illustrating the "preventive maintenance" character of this parameter (similar to α) mentioned above.

When λ_u and/or T_r is increased, $p_{d_{t_{c_1}}}$ has a stronger effect, as with α. This is shown in Figures B-28 and B-29. Cross sections of the curves of Figure B-28 for fixed T_s are shown in Figure B-20. This figure shows that, if T_s is sufficiently large, it is always best if the unit fails during checkout before the point of test decision, assuming there is a reasonable chance of detecting the failure.

The parameter $p_{d_{t_{c_2}}}$ is similar to μ_1, in that it directly affects the probability of entering standby good. However, as shown in Figure B-31, the optimum T_s increases as $p_{d_{t_{c_2}}}$ decreases, whereas it remained unchanged with μ_1 (as shown previously in Figures B-15 and B-17).

5. <u>Variable T_s, Parameters T_c and T_r (Figures B-32 to B-35)</u>

The increase in optimum standby time with increasing checkout and repair time is shown in Figures B-32 and B-33. Cross sections of these curves for fixed T_s, Figures B-34 and B-35, show that as standby time increases, the duration of the checkout and/or repair period becomes less and less significant, as expected. This is the opposite effect of that found for the parameters λ_s and and λ_u.

6. <u>Variable α, Parameters λ_u, $p_{u_{t_c}}$, μ_2 and T_r (Figures B-36 to B-40)</u>

This series of figures shows in more detail the changing relationships between P_{AR} and α, depending on the frequency of occurrence of undetectable failures. In Figures B-36 and 37, which have different values of T_s, the parameter is λ_u. For the values of λ_u shown, an optimium value of α exists. For small values or large values of λ_u, not shown in Fig. B-36, it may be best never to replace units which are not detectably bad, or always to replace them, respectively. The next figure (B-38) illustrates a similar behavior as $p_{u_{t_c}}$ varies, except that the optimum α is almost always 0 or 1. The curves all meet at $\alpha = 1$, since the value of $p_{u_{t_c}}$ is immaterial if the unit is always replaced.

An interesting series of curves results (Figure B-39) when μ_2 is varied, keeping μ_1 fixed. The quantity $1 - \mu_1 - \mu_2$ is the probability of undetectable failures occurring in the repaired/replaced unit. In the figure, μ_1 is held constant at 0.6, meaning 40 percent of the time a unit comes out of repair it will be in a failed condition. The top curve is for $\mu_2 = 0.4$, which means that none of the failures in repair are undetectable; and the bottom curve is for $\mu_2 = 0$, meaning all of the failures are undetectable. The curves in between represent the case where some of the failures are detectable. Some of these intermediate curves exhibit a maximum P_{AR} for $0 < \alpha < 1$. The curves cross at the point where $\alpha = 1 - \beta$ (i.e., $\alpha = 0.9$). This point represents the unrealistic situation where the unit is equally likely to be judged bad (during checkout) regardless of its actual condition. Thus, units coming out of repair with undetectable failures experience the same treatment at the next checkout as units with detectable failures—both have a 90 percent probability of entering repair. For values of α larger than 0.9, for example $\alpha = 1$, it is interesting to note that it is best to have all failures occurring during repair be undetectable! The reason for this is that units leaving repair with detectable failures may not be repaired at the next checkout, as there is a probability of β that the failure will not be detected. Units with only undetectable failures, however, will always enter repair, if they do not fail detectably before the next test decision.

Figure B-40 shows the changing slope of P_{AR} with α as repair time increases, discussed previously (also, see paragraph 8 below).

7. Variable $p_{dt_{c1}}$, Parameters p_{ut_c}, μ_2 and T_r (Figures B-41 to B-43)

These figures, when compared with Figures B-38 to B-40, illustrate the similarity mentioned earlier between the effects of $p_{dt_{c1}}$ and α, the value of $p_{dt_{c1}} = 0$ corresponding to $\alpha = 1$, $p_{dt_{c1}} = 1$ to $\alpha = 0$. Below, these similarities are discussed further.

8. Optimum Replacement Period (Figures B-44 to B-46)

Some types of units cannot be checked out, or it may be too costly or time-consuming to check them out. This is the reason the parameters λ_u, $p_{u_{t_c}}$, and μ_2 appear in the equation for alert readiness. An entire missile cannot be checked out completely until it is fired, and therefore may accumulate undetectable failures and should be replaced periodically. The optimum period between replacements for a unit not subject to checkout can be investigated by setting $\alpha = 1$, and $\beta = 0$. This means that at every time interval T_s the unit is replaced regardless of condition. For convenience, T_c and λ_s can be set equal to 0, since they are not distinguishable from T_r and λ_u, in this case. Since there is no checkout, many of the parameters are immaterial; the only ones of interest are T_s, T_r (or T_c), λ_u (or λ_s), and μ_1. Figures B-44 and B-46 show P_{AR} as a function of T_s for different values of λ_u, T_r, and μ_1, respectively. The optimum replacement time is seen to decrease significantly as λ_u increases or T_r decreases.

In discussing α and $p_{d_{t_{c_1}}}$ in paragraphs 6 and 7 above, it was found that there are many cases where P_{AR} does not have its maximum value at $\alpha = 0$ or $p_{d_{t_{c_1}}} = 1$. This can be interpreted to mean that in these cases there is an optimum replacement period for apparently good items, and the maintenance policy consists of concurrent optimum checkout and replacement periods. The simplest case is when $\alpha = 1$ or $p_{d_{t_{c_1}}} = 0$ gives the maximum P_{AR}, since this means the units should always be replaced after a standby period of T_s. Or, if the maximum occurs when $\alpha = 0$ or $p_{d_{t_{c_1}}} = 1$, the unit should never be replaced unless the checkout specifies it as bad. What if the maximum occurs at some intermediate value, say $\alpha = 1/2$ or $p_{d_{t_{c_1}}} = 1/2$? It is interesting to speculate as to whether this type of information could be used to arrive at an optimum maintenance policy.

We will assume that the actual checkout and test procedures themselves are completely specified, so that the question of arriving at an optimum policy refers only to the times at which checkout and replacement occur. With the checkout procedures

specified, there can be assumed to be an actual α, β, $p_{dt_{c_1}}$, etc., characteristic of the test, whose values will be assumed to be known or to have been estimated. If the actual $\alpha = 0.1$, say, and the optimum $\alpha = 0.5$, P_{AR} should increase if, on a random basis, more items are called bad during checkout than would normally be the case. This method will also affect β, however, since it is not known for sure during checkout that a given item is really good or bad. However, β will decrease in the above procedure, since more bad items will also be rightly called bad. Both effects increase alert readiness. Such a randomizing procedure could not be used, however, if the actual α were larger than the optimum; although α could be reduced (to 0 if necessary) by calling fewer items bad, β would increase and adversely affect alert readiness.

It should be remarked that this procedure has the effect of changing α; but it has not been shown that a specific, desired value of α (and a corresponding β) can be arrived at this way.

More useful and realistic than such a randomizing procedure, however, would be a periodic replacement of "good" units which, combined with the actual α (or $p_{dt_{c_1}}$), results in an α (or $p_{dt_{c_1}}$) which is near optimum. For example, if actual $\alpha = 0$, optimum $\alpha = 0.5$, instead of randomly replacing half the good units at each checkout period, each unit which experiences no detectable failures during two standby-checkout cycles could be replaced. This would have the same effect of half the "good" units being replaced and would result in a higher P_{AR} than the random method, since the undetectable failures accumulate with time, and no unit would be left unreplaced for more than two standby periods. If the optimum number of standby periods before automatic replacement turns out to be a nonintegral number between k and $k + 1$, then a randomizing procedure could be used to replace part of the items after k cycles and all the remaining items after $k + 1$ cycles.

REFERENCES

1. Monroe, A. J., "Missile/Launch Complex As-A-Unit Models for the Determination of Weapon System Capability," 6301-6237-TU-001, revised 7 March 1963, Space Technology Laboratories, Inc.

2. Monroe, A. J., "Weapon System Capability: Availability Models and Parameter Estimation," May 1963.

3. Kaufman, N., "Indirect Parameter Estimation," 6301-6438-RU-000, 27 April 1962, Space Technology Laboratories, Inc.

4. Kaufman, N., "Parameter Estimation for Continuously Monitored Systems," 6301-6280-KU-000, 23 July 1962, Space Technology Laboratories, Inc.

5. Kaufman, N., "Weapon System Availability Models and the Estimation of Their Parameters," Proceedings ASQC Long Island Quality Control and Reliability Conference, 20 April 1963.

6. Monroe, A. J., "Further Contributions to the Estimation of Parameters for Discretely Monitored Unit-Systems," 6101-7999-RU-000, 17 June 1963, Space Technology Laboratories, Inc.

7. Brooks, J. D., J. W. Green, D. A. Pope and J. F. Zancanaro, "The Maximization of Likelihood Functions," PA 4109-2, 11 March 1963, Space Technology Laboratories, Inc.

8. Monroe, A. J., "Generalized Unit Model for Equipment Availability Utilizing an Almost-Periodic Maintenance Cycle," 6301-6438-RU-000, 5 April 1963, Space Technology Laboratories, Inc.

APPENDIX A

TABLES OF PARAMETER ESTIMATES FROM MONTE CARLO RUNS

Table A-1: $p_s = 1.0$, $\alpha = 0.1$, $E = 1.0, 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	1.0	1.0	1.0	0.8	1.0	0.1	.9647	.4981	.1058	.0873
50	500							1.0807	.4994	.1002	.0768
100	500							1.0784	.1562	.1001	.0242
300	500							1.0078	.0149	.0973	.0053
500	300							1.0028	.0081	.0995	.0032
1000	100							1.0047	.0035	.1056	.0015
3000	50							1.0082	.0011	.0991	.0004
5000	50							.9991	.0006	.1068	.0003
10000	20							1.0013	.0004	.1026	.0001
25	500	0.9	1.0	1.0	0.8	1.0	0.1	.8335	.6786	.1362	.3938
50	500							.9432	.3970	.1286	.3669
100	500							.9532	.3375	.0168	.2552
300	500							.9105	.0230	.0925	.0195
500	300							.9064	.0107	.1048	.0104
1000	100							.8982	.0068	.0986	.0047
3000	50							.9060	.0014	.1034	.0013
5000	50							.8989	.0009	.0969	.0007
10000	20							.8981	.0004	.1064	.0005
25	500	0.7	1.0	1.0	0.8	1.0	0.1	.5850	.5700	.1561	.5120
50	500							.6864	.7832	.1504	.6739
100	500							.7155	.5742	.0849	.6026
300	500							.7321	.0625	.0553	.4254
500	300							.7194	.0269	.0961	.1992
1000	100							.7084	.0118	.0898	.0163
3000	50							.7155	.0031	.0971	.0028
5000	50							.6923	.0014	.1111	.0028
10000	20							.7025	.0003	.1010	.0009

Table A-2: $p_s = 1.0,\ \alpha = 0.3,\ E = 1.0,\ 0.9,\ 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	1.0	1.0	1.0	0.8	1.0	0.3	.8788	.8621	.2316	.3747
50	500							1.0509	1.0122	.3227	.4493
100	500							1.0818	1.2034	.2950	.4396
300	500							1.0885	.2488	.3152	.0673
500	300							1.0262	.0353	.3010	.0192
1000	100							.9895	.0119	.3304	.0082
3000	50							1.0008	.0040	.2988	.0019
5000	50							.9979	.0026	.3000	.0018
10000	20							1.0054	.0009	.3007	.0008
25	500	0.9	1.0	1.0	0.8	1.0	0.3	.7524	.9288	.3162	.4272
50	500							.8122	1.0127	.2850	.6024
100	500							.9778	1.2113	.3437	.9668
300	500							1.0312	.7569	.2480	.2929
500	300							.9318	.0444	.2938	.0575
1000	100							.9020	.0148	.2991	.0152
3000	50							.9173	.0056	.2967	.0044
5000	50							.9224	.0039	.3007	.0021
10000	20							.9058	.0012	.2966	.0015
25	500	0.7	1.0	1.0	0.8	1.0	0.3	.5637	.8086	.3663	.6862
50	500							.5362	1.3866	.3421	.8203
100	500							.6922	1.4771	.2929	1.0629
300	500							.7038	1.6509	.3091	1.5702
500	300							.7018	1.7154	.3104	.7782
1000	100							.7459	.0523	.3599	.8266
3000	50							.7348	.0146	.2843	.0163
5000	50							.6997	.0050	.2938	.0115
10000	20							.7147	.0027	.2959	.0033

<u>Table A-3</u>: $p_s = 1.0$, $\alpha = 0$, $E = 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	0.9	1.0	1.0	0.8	1.0	0	.8987	.0441	-.0324	.1960
50	500							.9033	.0325	-.0482	.2815
100	500							.8978	.0096	-.0498	.1023
300	500							.8990	.0025	-.0148	.0097
500	300							.8962	.0018	-.0084	.0063
1000	100							.9008	.0009	-.0069	.0031
3000	50							.9016	.0002	.0039	.0004
5000	50							.9007	.0002	-.0040	.0005
10000	20							.9002	.0001	.0009	.0003
25	500	0.7	1.0	1.0	0.8	1.0	0	.7149	.2896	.0777	.4929
50	500							.7032	.1988	-.0199	.5409
100	500							.7088	.0710	-.0796	.5609
300	500							.7070	.0100	-.0374	.0430
500	300							.6991	.0067	-.0262	.0195
1000	100							.7007	.0028	-.0187	.0066
3000	50							.6968	.0018	-.0052	.0019
5000	50							.6950	.0006	.0078	.0017
10000	20							.7000	.0003	.0027	.0005

Table A-4: $p_s = 0.8$, $\alpha = 0.1$, $E = 1.0, 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	1.0	1.0	0.8	0.8	1.0	0.1	.9144	.3875	.0876	.0648
50	500							1.0465	.4509	.0976	.0815
100	500							1.0507	.1798	.0985	.0352
300	500							1.0202	.0275	.1005	.0070
500	300							1.0060	.0113	.0960	.0039
1000	100							1.0013	.0054	.1075	.0015
3000	50							.9937	.0020	.0972	.0005
5000	50							.9961	.0008	.0992	.0004
10000	20							.9910	.0002	.1027	.0001
25	500	0.9	1.0	0.8	0.8	1.0	0.1	.8229	.3938	.2059	.3501
50	500							.8974	.4502	.1615	.5286
100	500							.9700	.5623	.0772	.4874
300	500							.9289	.0383	.0491	.1575
500	300							.9092	.0150	.0664	.0268
1000	100							.9025	.0043	.0735	.0093
3000	50							.9079	.0022	.1030	.0036
5000	50							.9035	.0019	.1067	.0015
10000	20							.8940	.0007	.0939	.0010
25	500	0.7	1.0	0.8	0.8	1.0	0.1	.5558	.5304	.2651	.5469
50	500							.6295	.8105	.1307	1.0090
100	500							.6960	.8254	.1549	1.0128
300	500							.7172	.2657	-.0982	1.5336
500	300							.7104	.1027	.0208	.4182
1000	100							.6856	.0210	.0556	.0508
3000	50							.6898	.0041	.1098	.0088
5000	50							.6950	.0027	.0881	.0042
10000	20							.6960	.0010	.1029	.0033

Table A-5: $p_s = 0.8$, $\alpha = 0.3$, $E = 1.0, 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	1.0	1.0	0.8	0.8	1.0	0.3	.7953	.6150	.2403	.4144
50	500							.8750	1.0350	.2779	.3866
100	500							1.0140	1.8519	.3042	.5478
300	500							1.1012	.2795	.3149	.1153
500	300							1.0260	.0511	.3045	.0321
1000	100							.9881	.0160	.3013	.0112
3000	50							1.0071	.0073	.2966	.0034
5000	50							1.0024	.0021	.3017	.0020
10000	20							1.0063	.0014	.2853	.0007
25	500	0.9	1.0	0.8	0.8	1.0	0.3	.6438	.7540	.3194	.7058
50	500							.7338	.9064	.3343	.8018
100	500							.8675	1.2236	.3133	1.2655
300	500							.9758	1.1367	.2585	.4000
500	300							.9216	.1718	.1886	.7532
1000	100							.9429	.0251	.2993	.0210
3000	50							.9012	.0073	.2942	.0094
5000	50							.9005	.0040	.3000	.0033
10000	20							.8991	.0020	.3143	.0019
25	500	0.7	1.0	0.8	0.8	1.0	0.3	.6117	.7509	.3497	.6386
50	500							.6099	.9960	.3781	.9618
100	500							.5213	1.6196	.3534	1.3567
300	500							.7007	1.2787	.3485	2.0369
500	300							.6494	.3742	.2692	1.4979
1000	100							.5848	1.6743	.2727	.1956
3000	50							.7094	.0176	.2256	.0562
5000	50							.6886	.0085	.3084	.0157
10000	20							.6999	.0045	.2931	.0086

Table A-6: $p_s = 0.8$, $\alpha = 0$, $E = 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	0.9	1.0	0.8	0.8	1.0	0	.8985	.0995	.0763	.4780
50	500							.9085	.0476	-.0401	.5289
100	500							.8893	.0224	-.1019	.3743
300	500							.8929	.0046	-.0216	.0348
500	300							.8961	.0027	-.0168	.0203
1000	100							.8941	.0014	-.0020	.0081
3000	50							.8944	.0003	-.0169	.0038
5000	50							.9010	.0003	-.0068	.0012
10000	20							.9023	.0002	-.0018	.0006
25	500	0.7	1.0	0.8	0.8	1.0	0	.7154	.3251	.2719	.6815
50	500							.6431	.3797	.1421	.7542
100	500							.6980	.2403	-.0215	1.2111
300	500							.6823	.0413	-.0655	.3612
500	300							.6995	.0134	-.1006	.2489
1000	100							.6995	.0091	-.0238	.0297
3000	50							.6855	.0023	.0114	.0068
5000	50							.6921	.0011	-.0084	.0056
10000	20							.7057	.0008	-.0050	.0016

Table A-7: $p_s = 0.5, \alpha = 0.1, E = 1.0, 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	1.0	1.0	0.5	0.8	1.0	0.1	.8448	.2894	.0595	.0813
50	500							.9930	.5920	.0831	.1140
100	500							1.1093	.6715	.1223	.0803
300	500							1.0259	.0468	.1037	.0134
500	300							1.0128	.0181	.1021	.0072
1000	100							.9858	.0073	.0945	.0027
3000	50							.9960	.0030	.0998	.0007
5000	50							.9940	.0015	.1010	.0006
10000	20							.9904	.0009	.0926	.0002
25	500	0.9	1.0	0.5	0.8	1.0	0.1	.6980	.4226	.3393	.6014
50	500							.8646	.6130	.3331	.8935
100	500							.8745	.4512	.1859	1.6403
300	500							.9421	.3199	-.0562	1.4992
500	300							.9221	.0555	.0316	.4287
1000	100							.9090	.0234	.0135	.1165
3000	50							.8968	.0036	.0972	.0126
5000	50							.9032	.0029	.0867	.0083
10000	20							.9064	.0008	.0837	.0033
25	500	0.7	1.0	0.5	0.8	1.0	0.1	.5864	.7653	.4185	.8639
50	500							.6335	1.1964	.4588	1.0096
100	500							.6863	1.1549	.3760	1.6753
300	500							.8494	2.1280	.1405	2.3053
500	300							.7103	.8417	.1103	2.5981
1000	100							.7068	.0894	.1374	.1945
3000	50							.6978	.0129	.1107	.0305
5000	50							.7052	.0076	.0827	.0230
10000	20							.6915	.0044	.1474	.0103

Table A-8: $p_s = 0.5$, $\alpha = 0.3$, $E = 1.0, 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	1.0	1.0	0.5	0.8	1.0	0.3	.6514	.4890	.2356	.2750
50	500							.8213	.8290	.2571	.3732
100	500							.9601	1.1517	.3080	.5285
300	500							1.0733	1.3732	.2994	.2921
500	300							1.0738	.3281	.3298	.0579
1000	100							1.0280	.0379	.3031	.0187
3000	50							1.0022	.0088	.2963	.0044
5000	50							1.0041	.0039	.2989	.0029
10000	20							1.0074	.0014	.2943	.0021
25	500	0.9	1.0	0.5	0.8	1.0	0.3	.7042	.5931	.4314	.4640
50	500							.7610	.8383	.4678	.8669
100	500							.7837	.8366	.4730	1.2840
300	500							.9222	.8295	.3799	2.1218
500	300							1.0811	1.6381	.2594	.6936
1000	100							.9213	.0490	.3920	1.5095
3000	50							.8967	.0095	.2811	.0259
5000	50							.8908	.0058	.2669	.0154
10000	20							.8955	.0035	.2916	.0045
25	500	0.7	1.0	0.5	0.8	1.0	0.3	.5396	.7220	.4552	.8485
50	500							.6622	1.2674	.4545	1.1414
100	500							.6870	1.7474	.4307	1.5534
300	500							.6952	2.3814	.4114	2.0217
500	300							.5537	2.9079	.4584	2.0662
1000	100							.8066	1.0651	.4366	1.6592
3000	50							.7297	.0901	.2554	.6355
5000	50							.7016	.0277	.2840	.0475
10000	20							.7117	.0062	.2315	.0379

Table A-9: $p_s = 0.5$, $\alpha = 0$, $E = 0.9, 0.7$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
25	500	0.9	1.0	0.5	0.8	1.0	0	.9144	.1388	.2945	.5800
50	500							.9053	.1565	.2249	.9957
100	500							.9233	.1202	.0340	1.4099
300	500							.9076	.0295	-.1362	.5464
500	300							.8989	.0073	-.1063	.1554
1000	100							.9035	.0029	-.0012	.0303
3000	50							.9008	.0010	.0125	.0129
5000	50							.8996	.0004	-.0061	.0073
10000	20							.9041	.0002	-.0043	.0029
25	500	0.7	1.0	0.5	0.8	1.0	0	.7530	.6021	.4686	.8165
50	500							.7148	.6840	.3347	1.2678
100	500							.7089	1.1675	.2600	1.7614
300	500							.6844	.7999	.0167	3.4586
500	300							.6458	.6273	-.0929	3.6239
1000	100							.6828	.0291	.1184	3.7966
3000	50							.6949	.0072	-.0752	.0744
5000	50							.7005	.0045	-.0240	.0193
10000	20							.6927	.0021	-.0598	.0171

Table A-10: $E = \mu = p_s = p_{c_2} = 1.0,\ p_{c_1} = 0.5$

								$\hat{E}$		$\hat{\alpha}$	
M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	Mean	Variance	Mean	Variance
100	500	1.0	1.0	1.0	0.5	1.0	0	1.0000	.0000	.0000	.0000
	500						.05	1.0047	.0025	.0535	.0055
	500						.10	1.0081	.0054	.1013	.0119
	500						.20	1.0155	.0113	.1872	.0427
500	500	1.0	1.0	1.0	0.5	1.0	0	1.0000	.0000	.0000	.0000
	500						.05	1.0011	.00004	.0512	.0008
	500						.10	1.0017	.0008	.1026	.0021
	500						.20	.9995	.0017	.2050	.0056
1000	100	1.0	1.0	1.0	0.5	1.0	0	1.0000	.0000	.0000	.0000
	100						.05	1.0020	.0002	.0531	.0005
	100						.10	1.0063	.0004	.0999	.0007
	100						.20	1.0004	.0009	.1998	.0024

Table A-11: $E = \mu = p_s = p_{c_1} = 1$, $p_{c_2} = 0.5$ (Favorable Case for α)

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
100	500	1.0	1.0	1.0	1.0	0.5	0	1.0000	.0000	.0000	.0000
	500						.05	1.0004	.0014	.0504	.0024
	500						.10	1.0008	.0020	.0997	.0059
	500						.20	1.0060	.0044	.2004	.0120
500	500	1.0	1.0	1.0	1.0	0.5	0	1.0000	.0000	.0000	.0000
	500						.05	.9989	.0002	.0503	.0004
	500						.10	1.0009	.0004	.1009	.0009
	500						.20	.9980	.0008	.2018	.0024
1000	100	1.0	1.0	1.0	1.0	0.5	0	1.0000	.0000	.0000	.0000
	100						.05	.9969	.0001	.0491	.0002
	100						.10	1.0000	.0002	.1022	.0006
	100						.20	.9998	.0004	.1973	.0009

Table A-12: $\mu = p_s = p_{c_1} = 1.0$, $E = 0.95, 0.9, 0.8$, $p_{c_2} = 0.2, 0.4, 0.6, 0.8$

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance
100 1000	500 50	0.95	1.0	1.0	1.0	0.2	0	.9501 .9489	.0007 .0001
100 1000	500 50	0.9						.8981 .8999	.0021 .0001
100 1000	500 50	0.8						.8028 .8011	.0025 .0002
100 1000	500 50	0.95	1.0	1.0	1.0	0.4	0	.9509 .9496	.0009 .0001
100 1000	500 50	0.9						.9014 .8974	.0015 .0002
100 1000	500 50	0.8						.8023 .7984	.0033 .0003
100 1000	500 50	0.95	1.0	1.0	1.0	0.6	0	.9513 .9499	.0012 .0001
100 1000	500 50	0.9						.8992 .8993	.0026 .0002
100 1000	500 50	0.8						.8021 .8037	.0050 .0004
100 1000	500 50	0.95	1.0	1.0	1.0	0.8	0	.9488 .9492	.0027 .0003
100 1000	500 50	0.9						.8949 .9492	.0046 .0003
100 1000	500 50	0.9						.8949 .8983	.0046 .0007
1[illegible] 100[illegible]	500 50	0.8						.7918 .8088	.[illegible]9 .[illegible]8

Table A-13: $\mu = p_s = p_{c_1} = 1.0$, $p_{c_2} = \alpha = 0$ (Favorable Case for $\hat{E}$)

M	N	E	μ	p_s	p_{c_1}	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance
50	500	0.95	1.0	1.0	1.0	0	0	.9495	.0010
100	500							.9507	.0005
500	500							.9492	.0001
1000	100							.9492	.0000
50	500	0.9	1.0	1.0	1.0	0	0	.8963	.0021
100	500							.9008	.0010
500	500							.9004	.0002
1000	100							.8991	.0001
50	500	0.8	1.0	1.0	1.0	0	0	.8002	.0042
100	500							.8013	.0020
500	500							.8012	.0004
1000								.7999	.0002

Table A-14: Comparison of Favorable $\hat{\alpha}$ Cases

$p_s = p_{c_1} = 1.0,\ p_{c_2} = 0.5$

M	N	E	μ	α	$\hat{\alpha}$ Mean	$\hat{\alpha}$ Variance
100	500	1.0	1.0	.05	.0504	.0024
		.9			.0483	.0038
		.8			.0476	.0074
		.7			.0516	.0368
		1.0	.9		.0517	.0027
			.8		.0499	.0030
			.7		.0467	.0037
500	500	1.0	1.0	.05	.0503	.0004
	300	.9			.0499	.0007
		.8			.0510	.0011
		.7			.0516	.0017
		1.0	.9		.0478	.0005
			.8		.0516	.0006
			.7		.0504	.0007
1000	100	1.0	1.0	.05	.0491	.0002
	50	.9			.0473	.0003
		.8			.0486	.0008
		.7			.0478	.0010
		1.0	.9		.0458	.0002
			.8		.0482	.0002
			.7		.0581	.0003
100	500	1.0	1.0	.10	.0997	.0059
		.9			.0995	.0098
		.8			.1011	.0220
		.7			.1144	.0783
		1.0	.9		.0984	.0059
			.8		.0977	.0060
			.7		.1099	.0089
500	500	1.0	1.0	.10	.1009	.0009
	300	.9			.0971	.0015
		.8			.0993	.0021
		.7			.0948	.0034
		1.0	.9		.1033	.0011
			.8		.1034	.0013
			.7		.1029	.0015

Table A-14 (Cont.)

M	N	E	μ	α	$\hat{\alpha}$ Mean	Variance
1000	100	1.0	1.0	.10	.1029	.0006
	50	.9			.1024	.0011
		.8			.1021	.0012
		.7			.1019	.0018
		1.0	.9		.1014	.0004
			.8		.1022	.0006
			.7		.0876	.0008
100	500	1.0	1.0	.20	.2004	.0120
		.9			.2055	.0300
		.8			.1870	.0778
		.7			.1845	.2506
		1.0	.9		.1964	.0177
			.8		.2002	.0180
			.7		.1923	.0257
500	500	1.0	1.0	.20	.2018	.0024
	300	.9			.2028	.0043
		.8			.1998	.0064
		.7			.1990	.0107
		1.0	.9		.2028	.0027
			.8		.2004	.0032
			.7		.1956	.0036
1000	100	1.0	1.0	.20	.1973	.0009
	50	.9			.1979	.0014
		.8			.2044	.0034
		.7			.1915	.0051
		1.0	.9		.1993	.0012
			.8		.2043	.0009
			.7		.1945	.0013

Table A-15: Comparison of Favorable $\hat{E}$ Cases

$p_s = p_{c_1} = 1.0$

M	N	E	μ	p_{c_2}	α	$\hat{E}$ Mean	Variance
100	500	.95	1.0	0	0	.9507	.0005
					.1	.9513	.0006
					.2	.9516	.0007
					.3	.9504	.0007
			.9		0	.9495	.0006
			.8			.9511	.0006
			.7			.9504	.0008
			1.0	.2		.9501	.0007
				.4		.9509	.0009
				.6		.9513	.0012
				.8		.9488	.0027
1000	100	.95		0	0	.9492	.0000
	50				.1	.9495	.0001
					.2	.9488	.0001
					.3	.9485	.0001
			.9		0	.9491	.0001
			.8			.9507	.0000
			.7			.9499	.0001
			1.0	.2		.9489	.0001
				.4		.9496	.0001
				.6		.9499	.0001
				.8		.9492	.0003
100	500	.90	1.0	0	0	.9008	.0010
					.1	.9005	.0011
					.2	.8982	.0013
					.3	.9007	.0015
			.9		0	.8986	.0013
			.8			.8985	.0013
			.7			.9014	.0015
			1.0	.2		.8981	.0012
				.4		.9014	.0015
				.6		.8992	.0026
				.8		.8949	.0046
1000	100	.90	1.0	0	0	.8991	.0001
	50				.1	.8990	.0001
					.2	.9009	.0001
					.3	.8986	.0001
			.9		0	.8971	.0001
			.8			.8996	.0001

Table A-15 (Cont.)

M	N	E	μ	p_{c_2}	α	$\hat{E}$ Mean	$\hat{E}$ Variance
			.7			.9009	.0001
			1.0	.2		.8999	.0001
				.4		.8974	.0002
				.6		.8993	.0002
				.8		.8983	.0007
100	500	.80	1.0	0	0	.8013	.0020
					.1	.7982	.0024
					.2	.7965	.0033
					.3	.7930	.0041
			.9		0	.8013	.0024
			.8			.7991	.0026
			.7			.8029	.0031
			1.0	.2		.8028	.0025
				.4		.8023	.0033
				.6		.8021	.0050
				.8		.7918	.0119
1000	100	.80	1.0	0	0	.7999	.0002
	50				.1	.8007	.0003
					.2	.8006	.0003
					.3	.8046	.0005
			.9		0	.7976	.0002
			.8			.8013	.0003
			.7			.7963	.0003
			1.0	.2		.8011	.0002
				.4		.7984	.0003
				.6		.8037	.0004
				.8		.8088	.0008

Table A-16: Estimation of All Parameters (Except p_{c_2}) and Availability

	$\hat{E}$	$\hat{\alpha}$	$\hat{p}_{c_1}$	$\hat{\mu}$	$\hat{p}_s$	$\hat{\lambda}_g$	$\hat{P}(G)$	$\hat{A}$ (T_s = 1, T_c = .012, T_r = .060)
1.	.8750	.2110	.8516	.498	1.200 (1.0)	(0)	.715	.689
2.	.8421	.1026	.5132	.679	1.100 (1.0)	(0)	.717	.678
3.	.8683	.1461	.5329	.813	.634	.455	.776	.593
4.	.8909	.0274	.4424	.830	.889	.118	.822	.738
5.	.9455	.1647	.5152	.896	.301	1.200	.865	.474
6.	.9286	-.2131 (0)	.3631	.828	.638	.450	.812	.618
7.	.9023	-.1159 (0)	.3966	.794	.558	.583	.762	.546
8.	.8927	.0690	.3277	1.103 (1.0)	.792	.233	.918	.779
9.	.9017	-.0755 (0)	.3064	1.032 (1.0)	.656	.421	.920	.774
10.	.8466	.0833	.3409	1.022 (1.0)	.880	.129	.887	.786
11.	.8994	.0000	.3520	1.038 (1.0)	.805	.217	.926	.791
12.	.8129	.2716	.4737	.952	1.218 (1.0)	(0)	.861	.820
13.	.9651	.0521	.5581	.787	.750	.288	.840	.694
14.	.9205	.2632	.6291	.841	.588	.530	.831	.612
15.	.8779	.1184	.4419	.816	.623	.473	.772	.583
16.	.8947	.2381	.6316	.765	.829	.188	.792	.687
17.	.8758	.2482	.6209	.737	.560	.580	.718	.516
18.	.8611	.2604	.6667	.723	.692	.369	.725	.575
19.	.8167	.2353	.3778	1.181 (1.0)	.708	.345	.859	.690
20.	.9459	.1548	.5676	.834	.596	.518	.842	.622
Average	.8882	.1139	.4955	.858	.750	.355	.818	.663
σ^2	.0016	.0183	.0188					
True Values	.9	.1	.5	.8	.7	.357	.792	.621

Note: M = 1000, p_{c_2} = 1

Values in parentheses were used in calculating $\hat{P}(G)$ and $\hat{A}$.

Table A-17. List of Symbols Appearing in Appendix A

T_r = Duration of repair period

T_s = Duration of standby period

T_{c_1} = Duration of checkout interval prior to test decision

T_{c_2} = Duration of checkout interval after test decision

D = Point (in time) of test decision

E = Probability that a unit which is failed at D will be declared bad at D

α = Probability that a unit which is good at D will be declared bad at D

μ = Probability that a unit is good at completion of repair

p_s = Probability that a unit which is good at entrance to standby is still good at completion of standby

p_{c_1} = Probability that a unit which is good at entrance to checkout is still good at D

p_{c_2} = Probability that a unit which is good at D is still good at completion of checkout

M = Number of units in sample

N = Number of computer runs with M units each

APPENDIX B

GRAPHS SHOWING RELATIONSHIP OF AVAILABILITY TO OPERATIONAL AND MAINTENANCE PARAMETERS

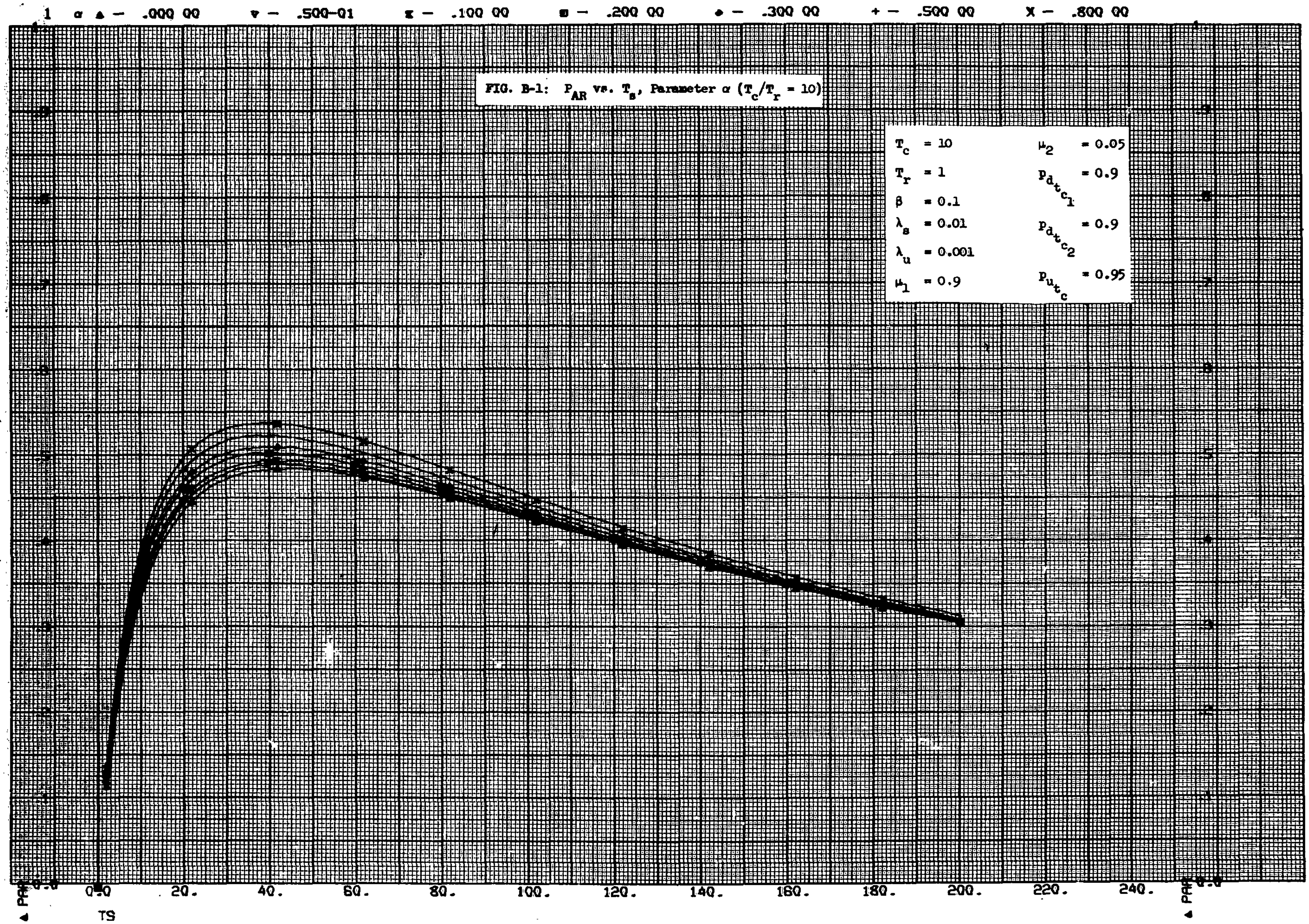

FIG. B-1: P_{AR} vs. T_s, Parameter α (T_c/T_r = 10)

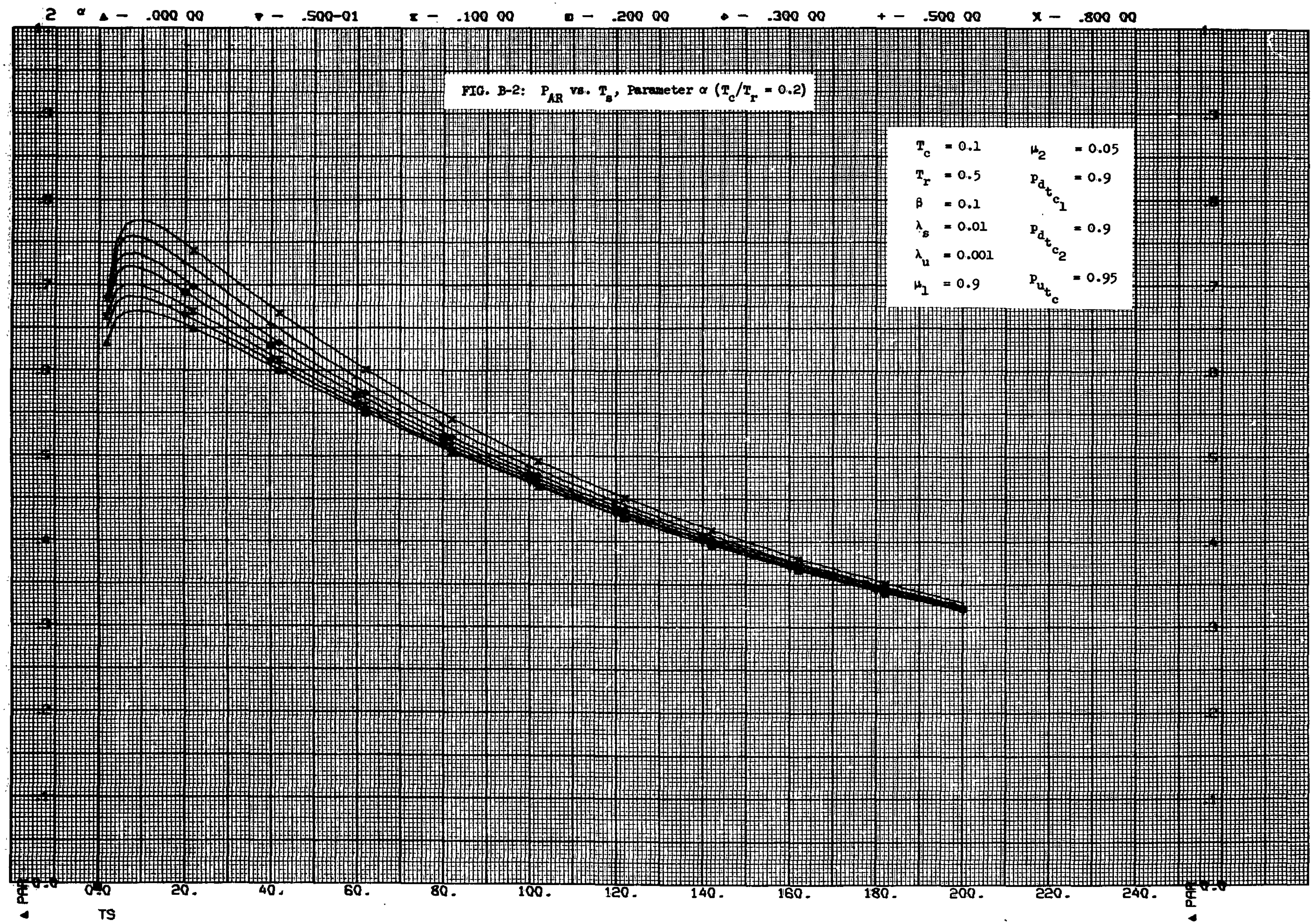

FIG. B-2: P_{AR} vs. T_s, Parameter α (T_c/T_r = 0.2)

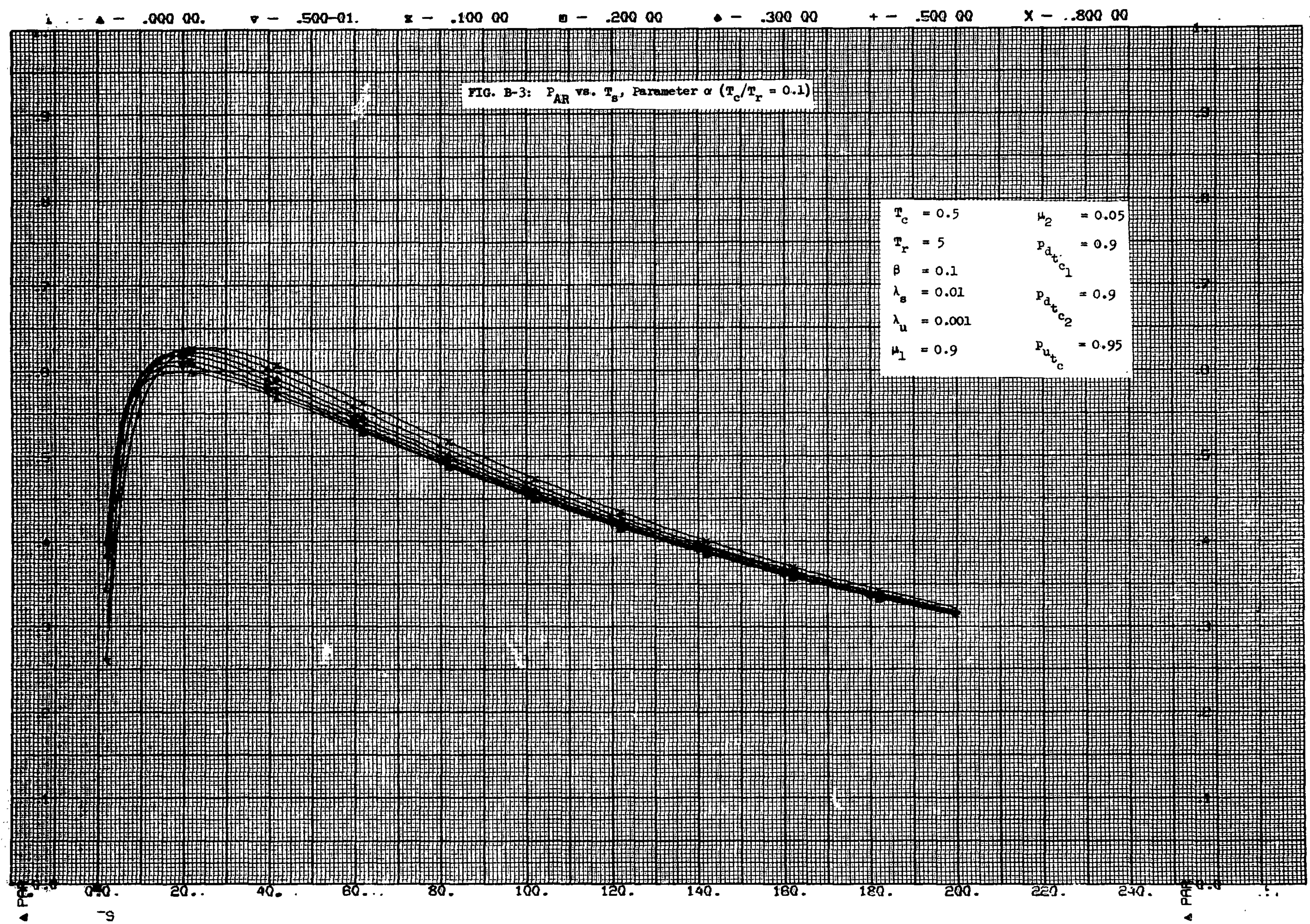

FIG. B-3: P_{AR} vs. T_s, Parameter α (T_c/T_r = 0.1)

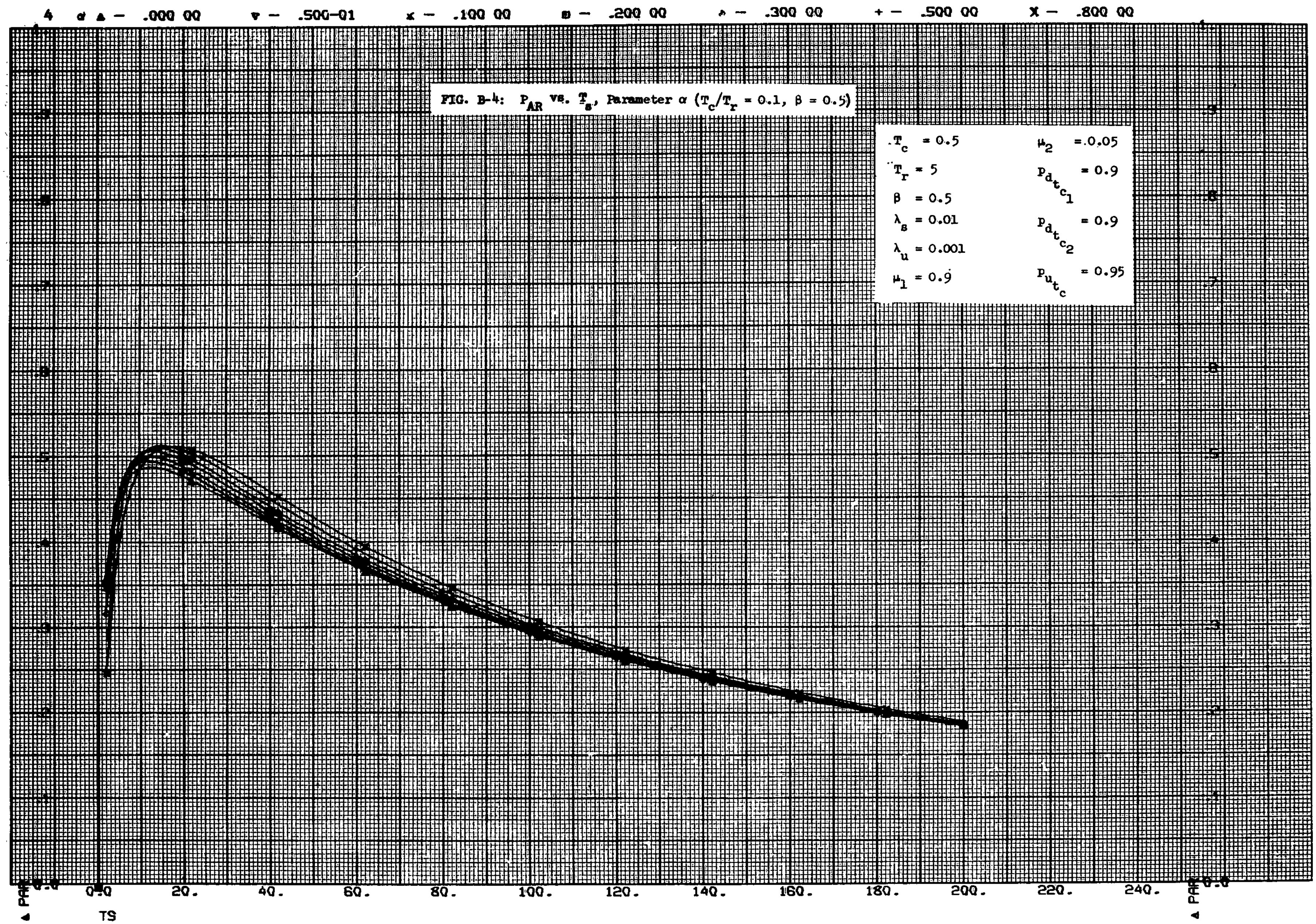

FIG. B-4: P_{AR} vs. T_s, Parameter α (T_c/T_r = 0.1, β = 0.5)

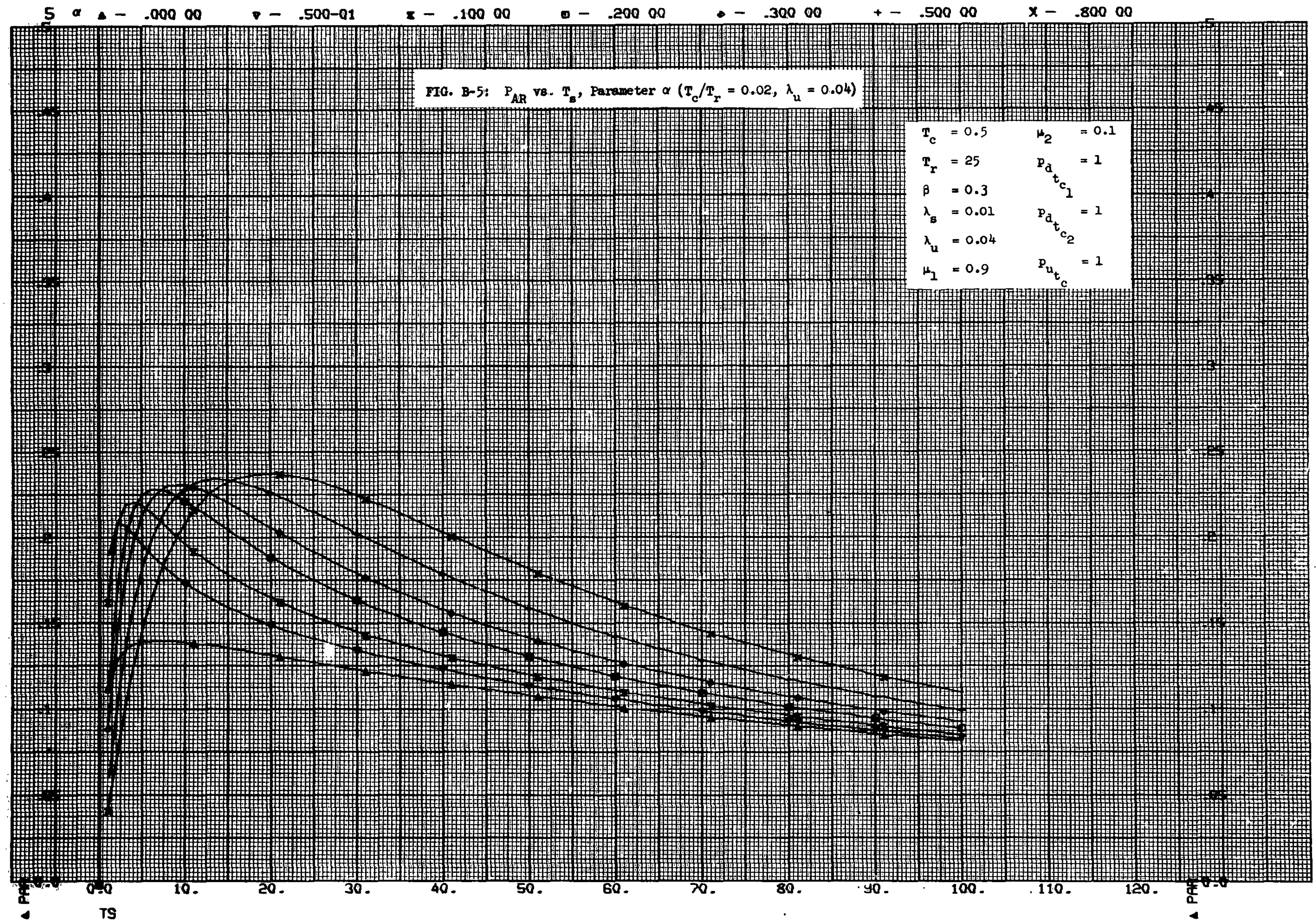

FIG. B-5: P_{AR} vs. T_s, Parameter α (T_c/T_r = 0.02, λ_u = 0.04)

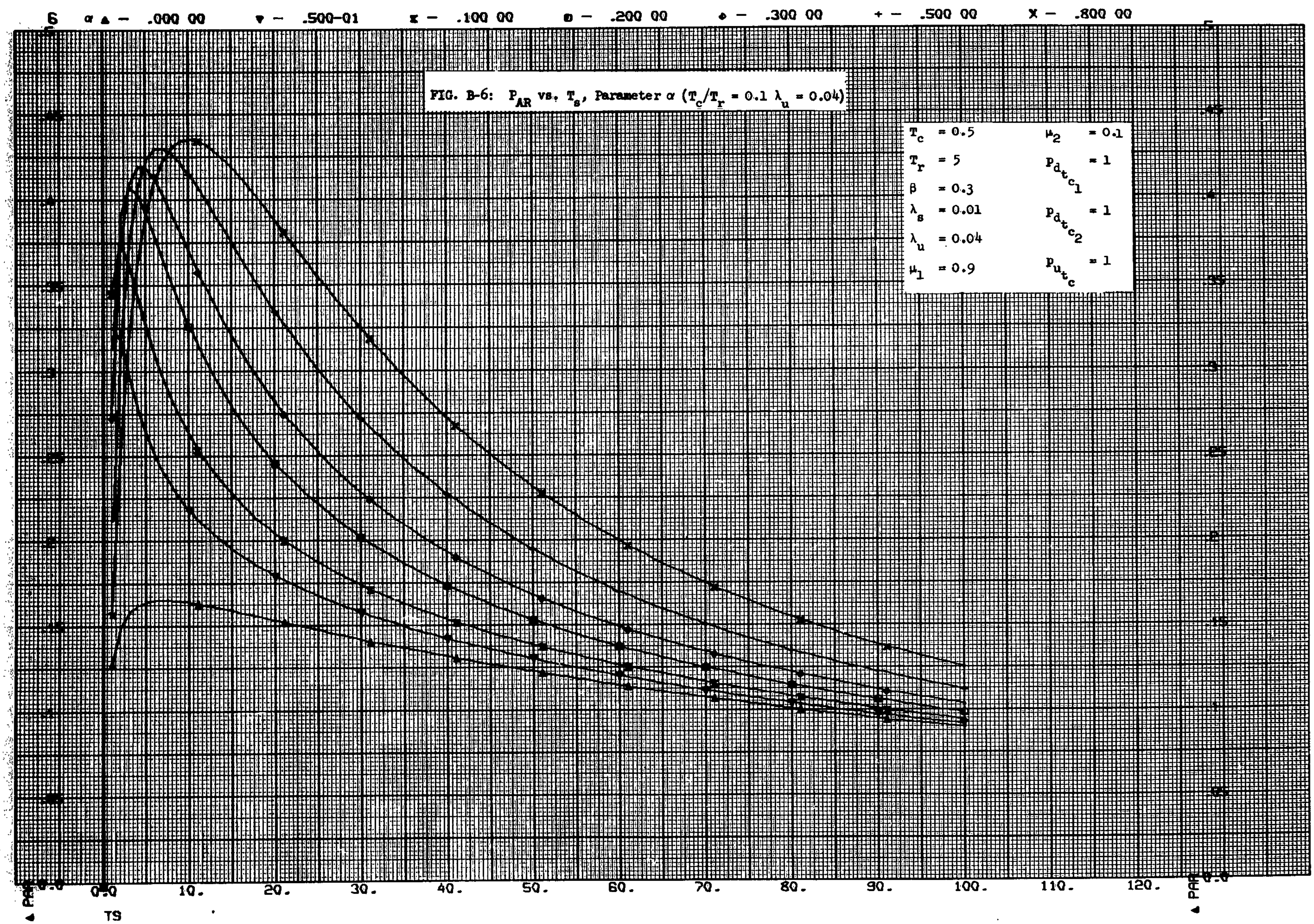

FIG. B-6: P_{AR} vs. T_s, Parameter α (T_c/T_r = 0.1 λ_u = 0.04)

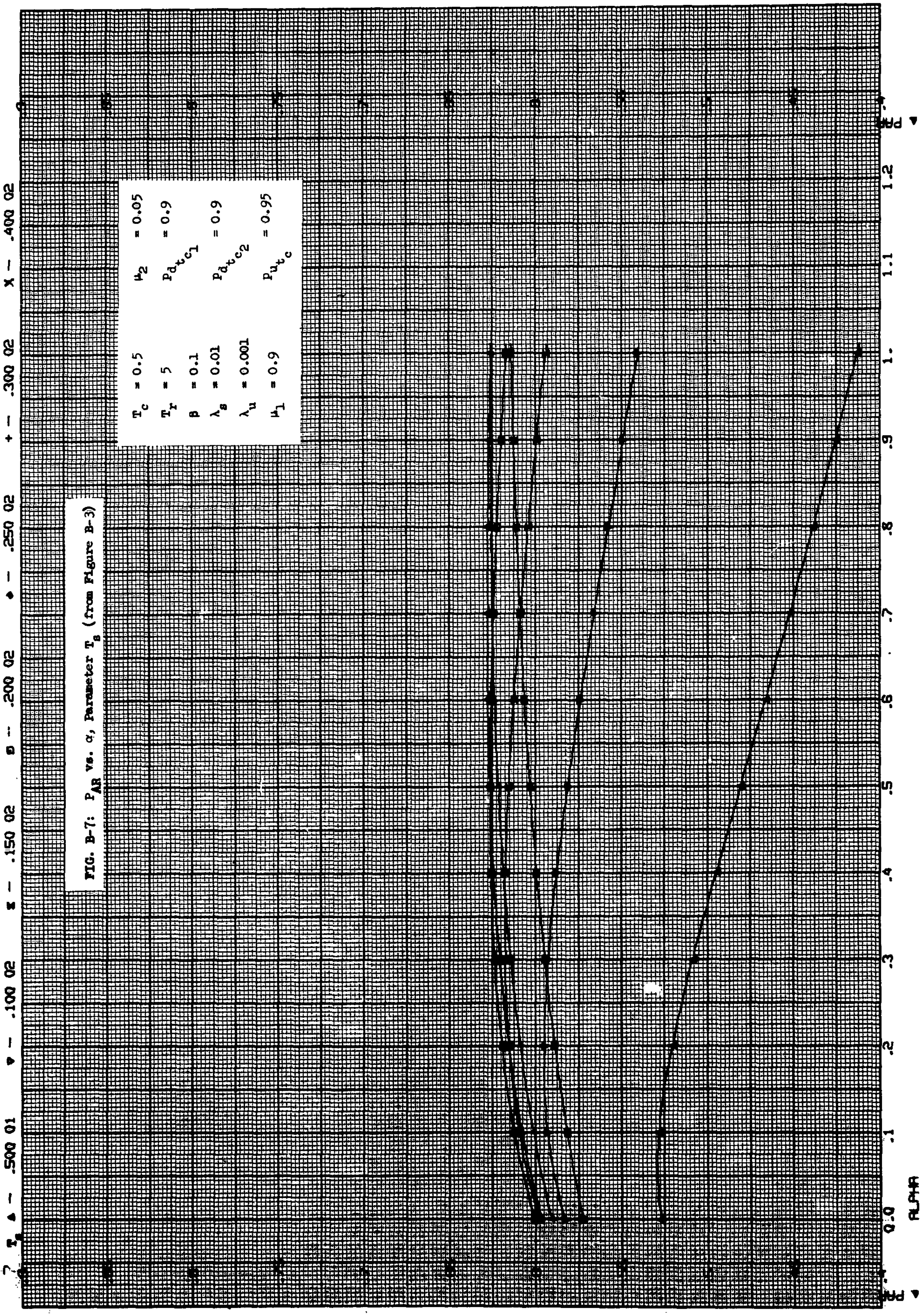

FIG. B-7: P_{AR} vs. α, Parameter T_s (from Figure B-3)

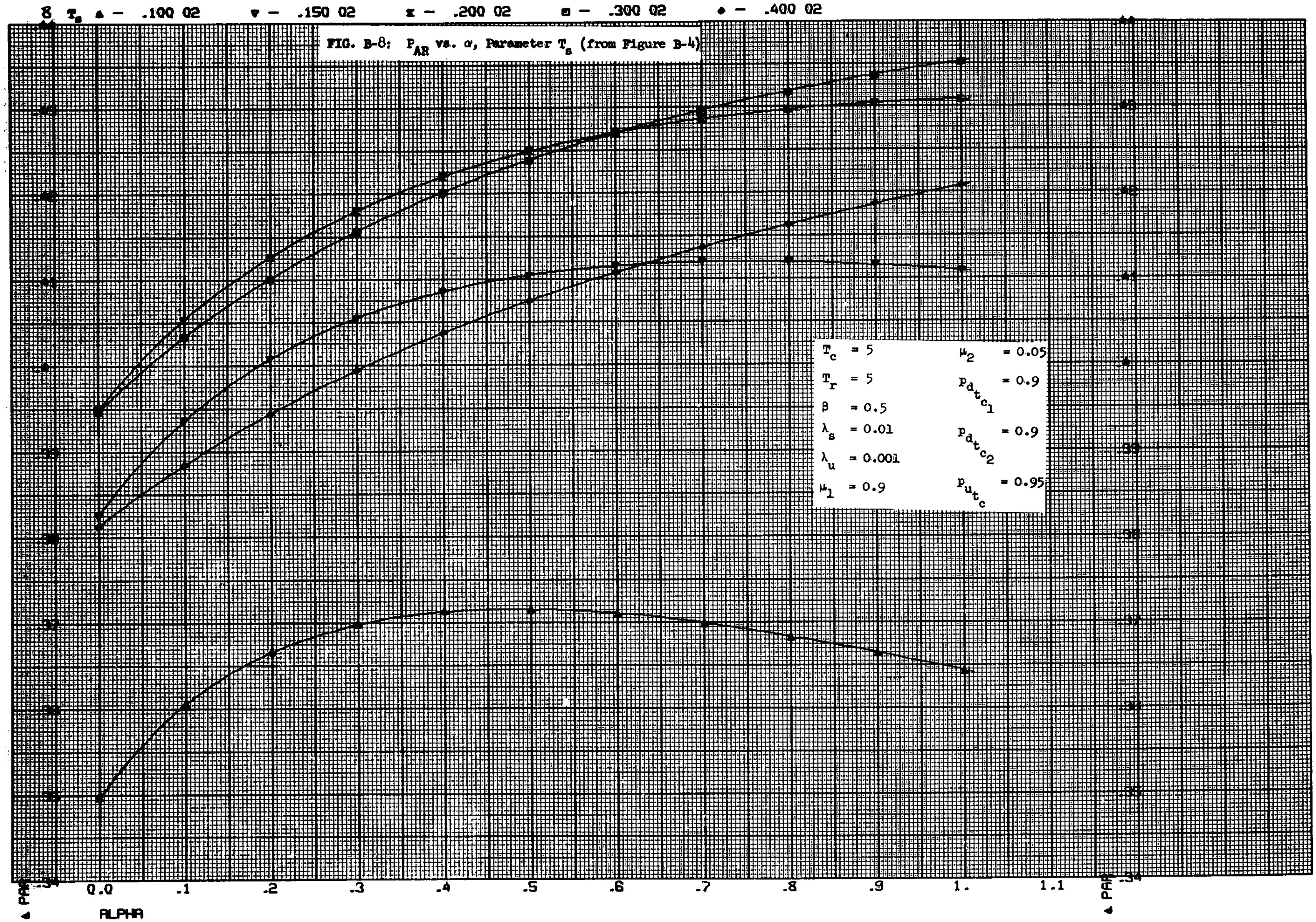

FIG. B-8: P_{AR} vs. α, Parameter T_s (from Figure B-4)

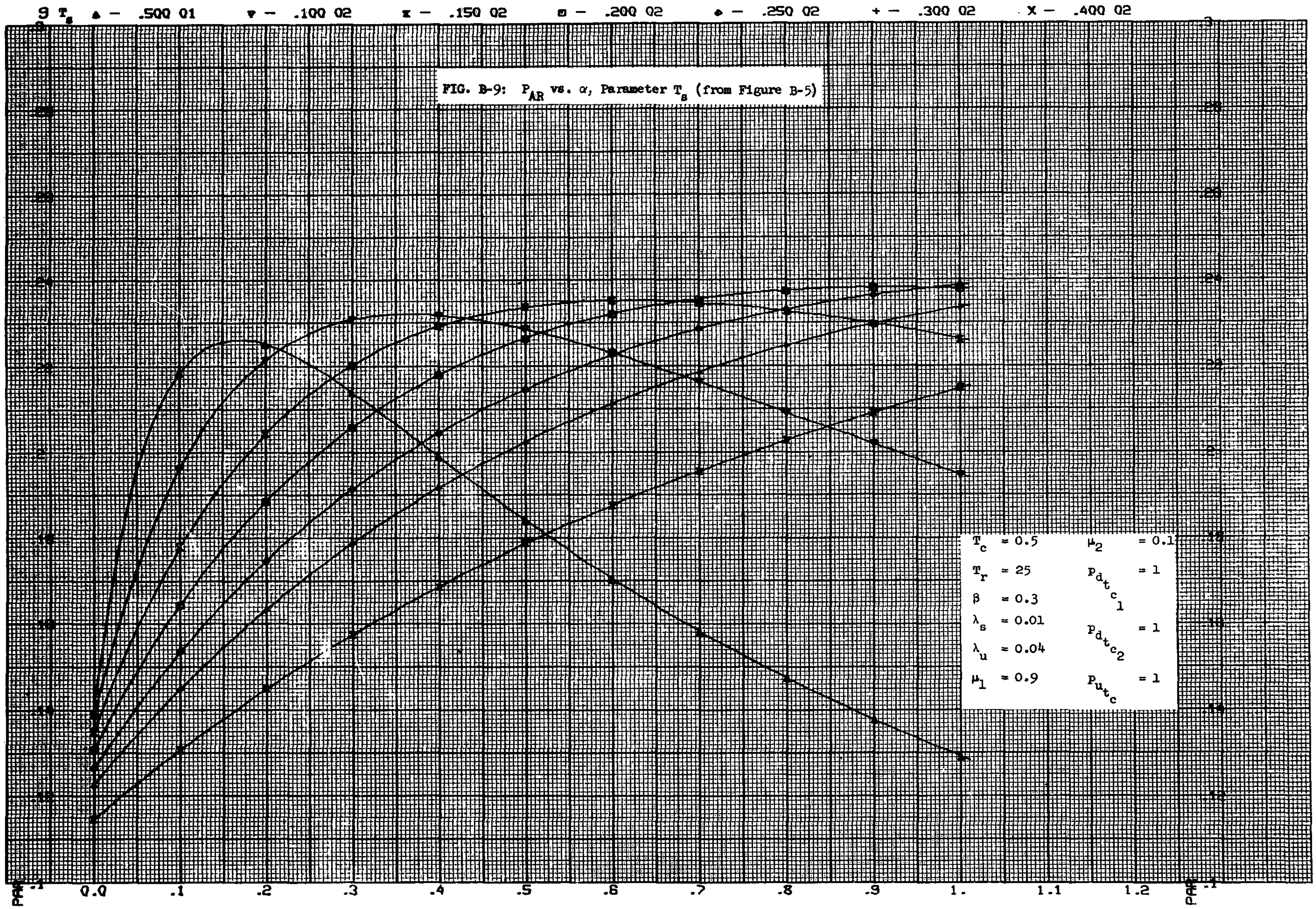

FIG. B-9: P_{AR} vs. α, Parameter T_s (from Figure B-5)

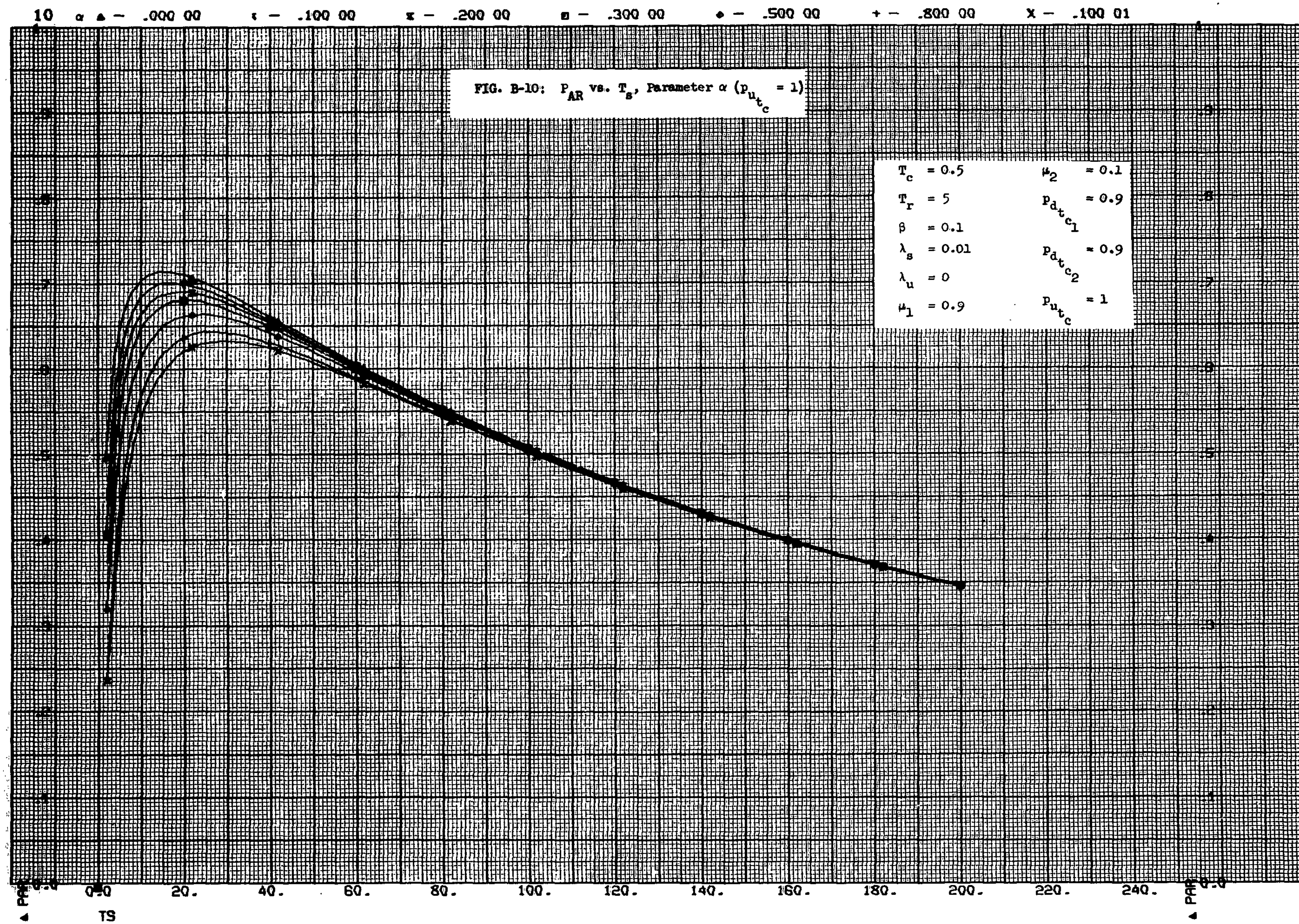

FIG. B-10: P_{AR} vs. T_s, Parameter α ($p_{u_{t_c}}$ = 1)

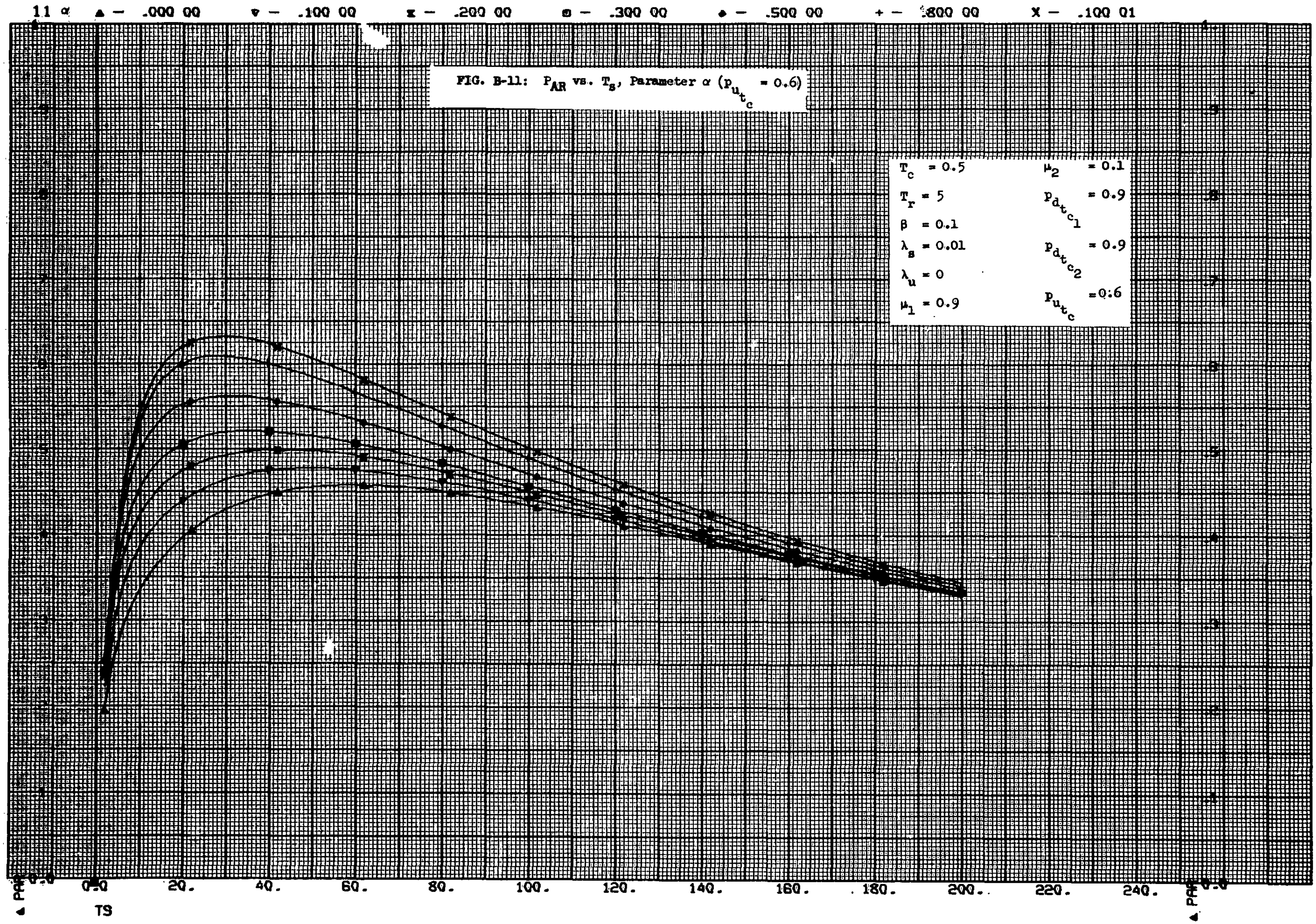

FIG. B-11: P_{AR} vs. T_s, Parameter α ($p_{u_{t_c}}$ = 0.6)

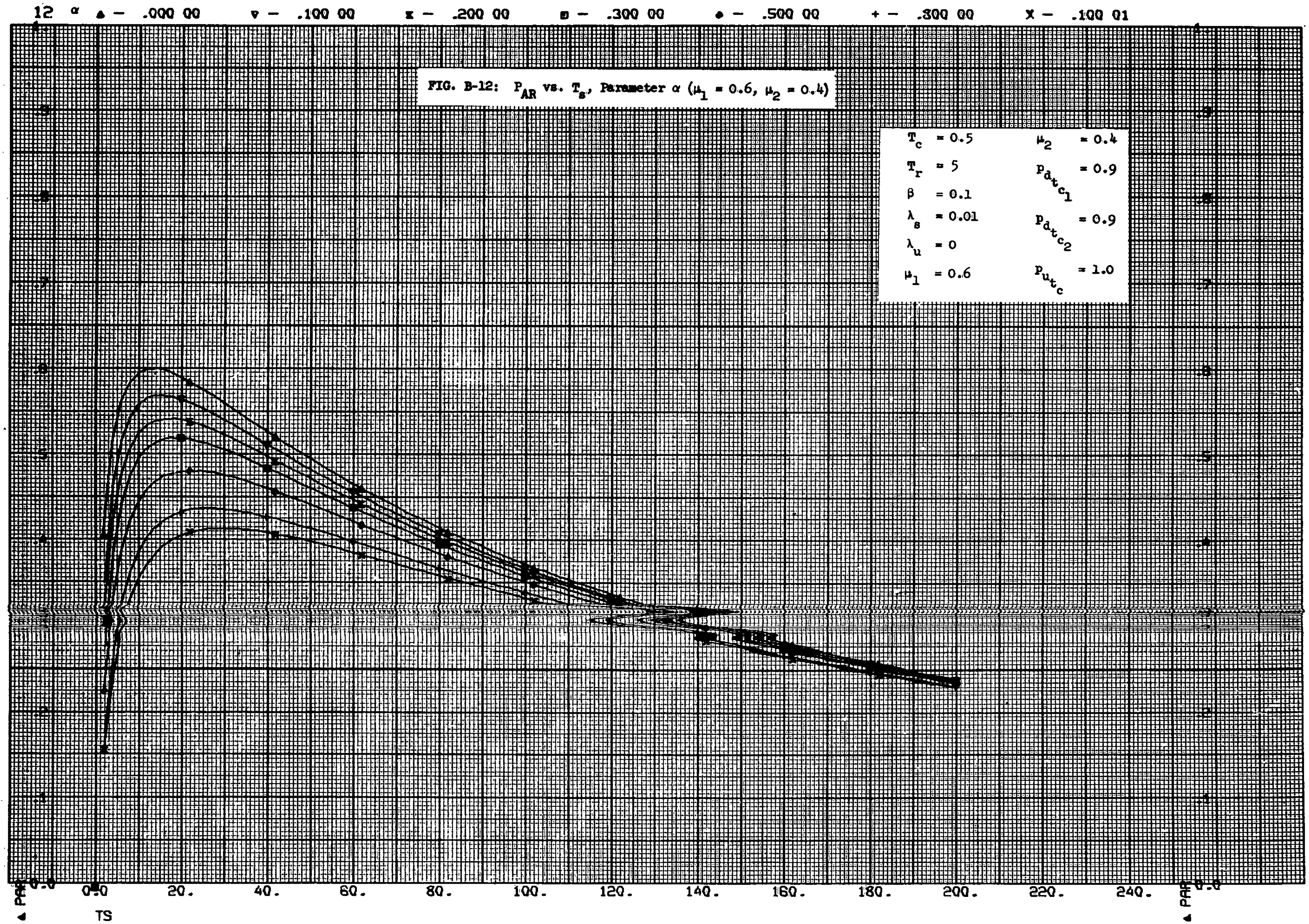

FIG. B-12: P_{AR} vs. T_s, Parameter α (μ_1 = 0.6, μ_2 = 0.4)

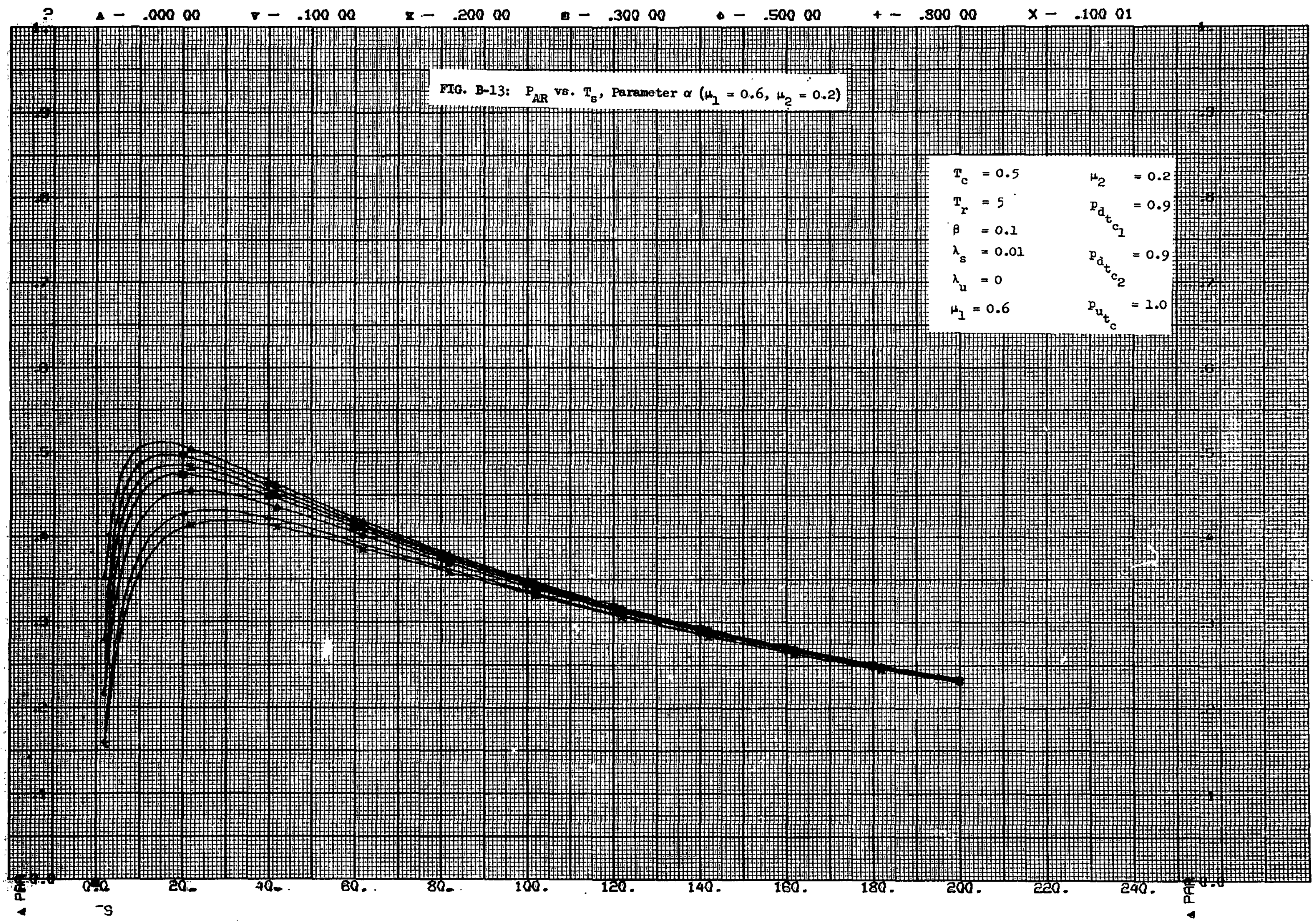

FIG. B-13: P_{AR} vs. T_s, Parameter α ($\mu_1 = 0.6$, $\mu_2 = 0.2$)

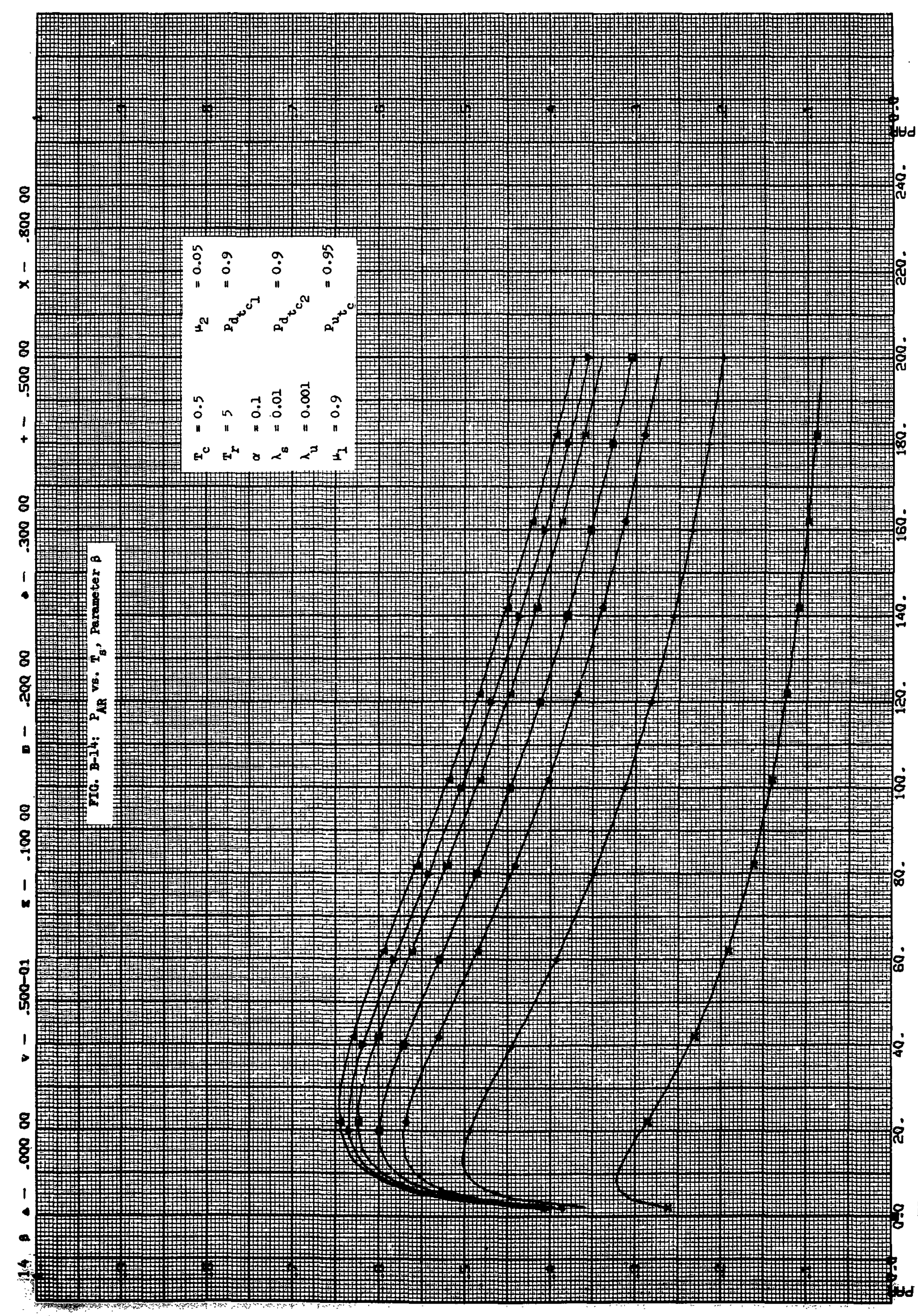

FIG. B-14: P_{AR} vs. T_s, Parameter β

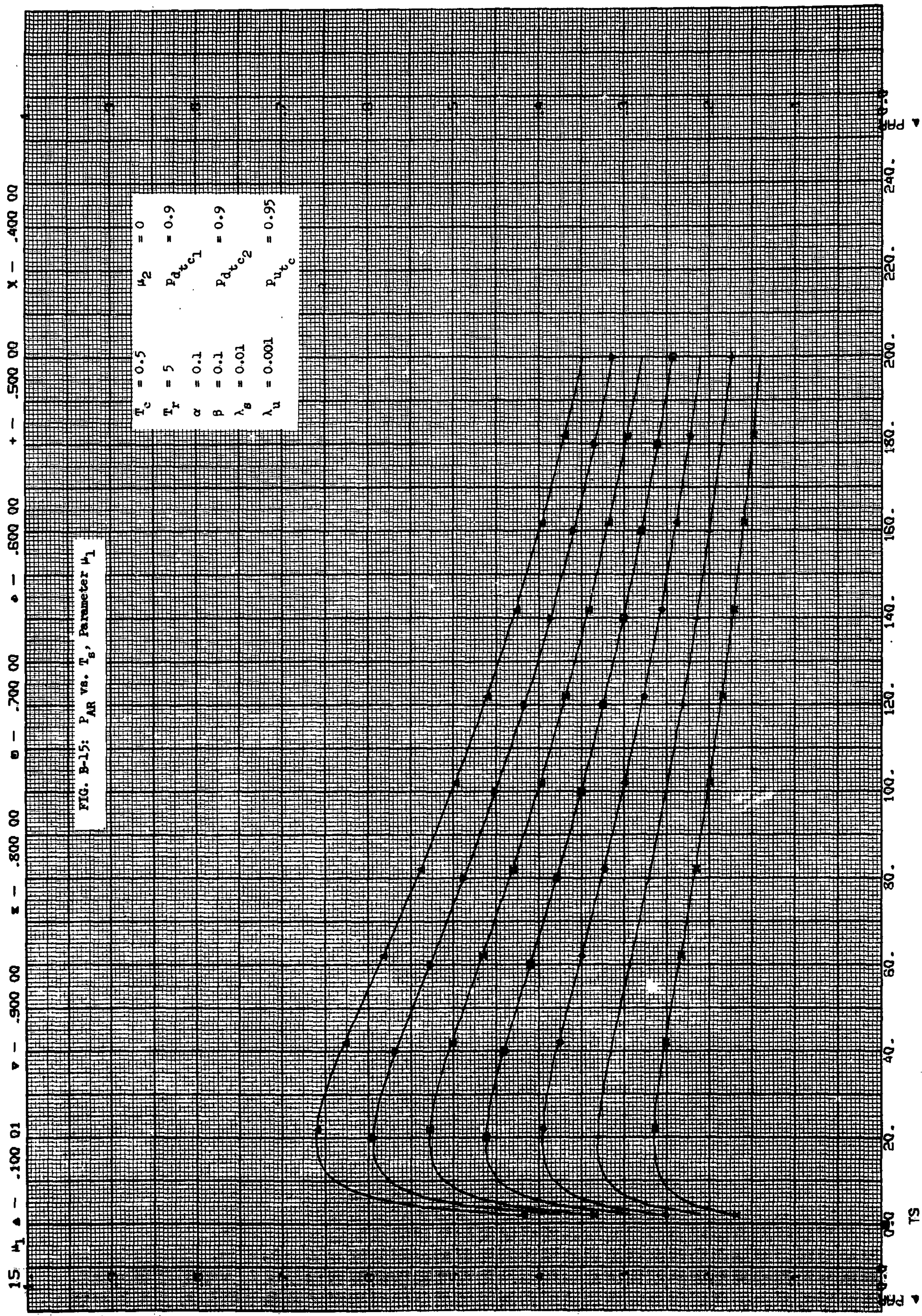

FIG. B-15: P_{AR} vs. T_s, Parameter μ_1

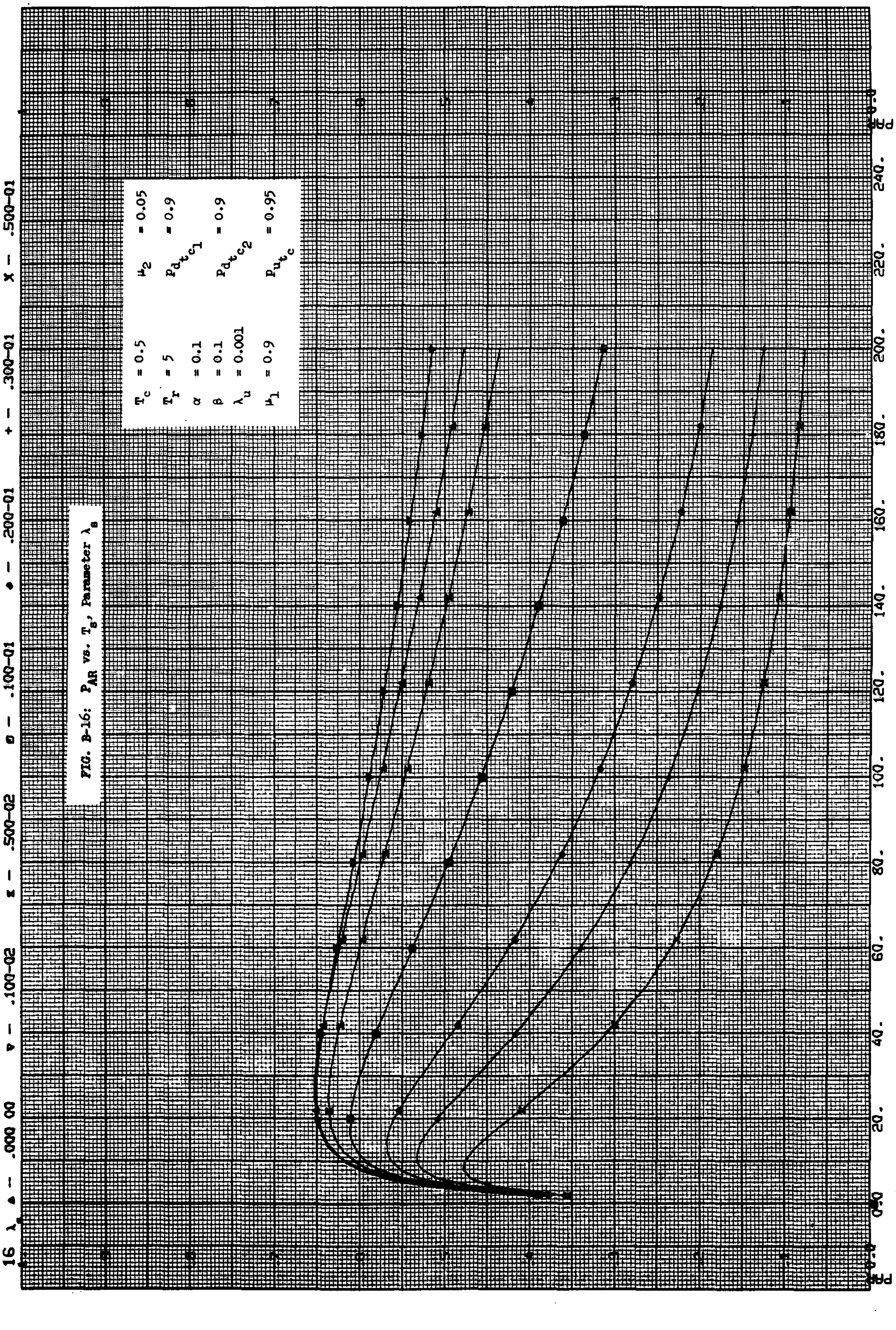

FIG. B-16: P_{AR} vs. T_s, Parameter λ_s

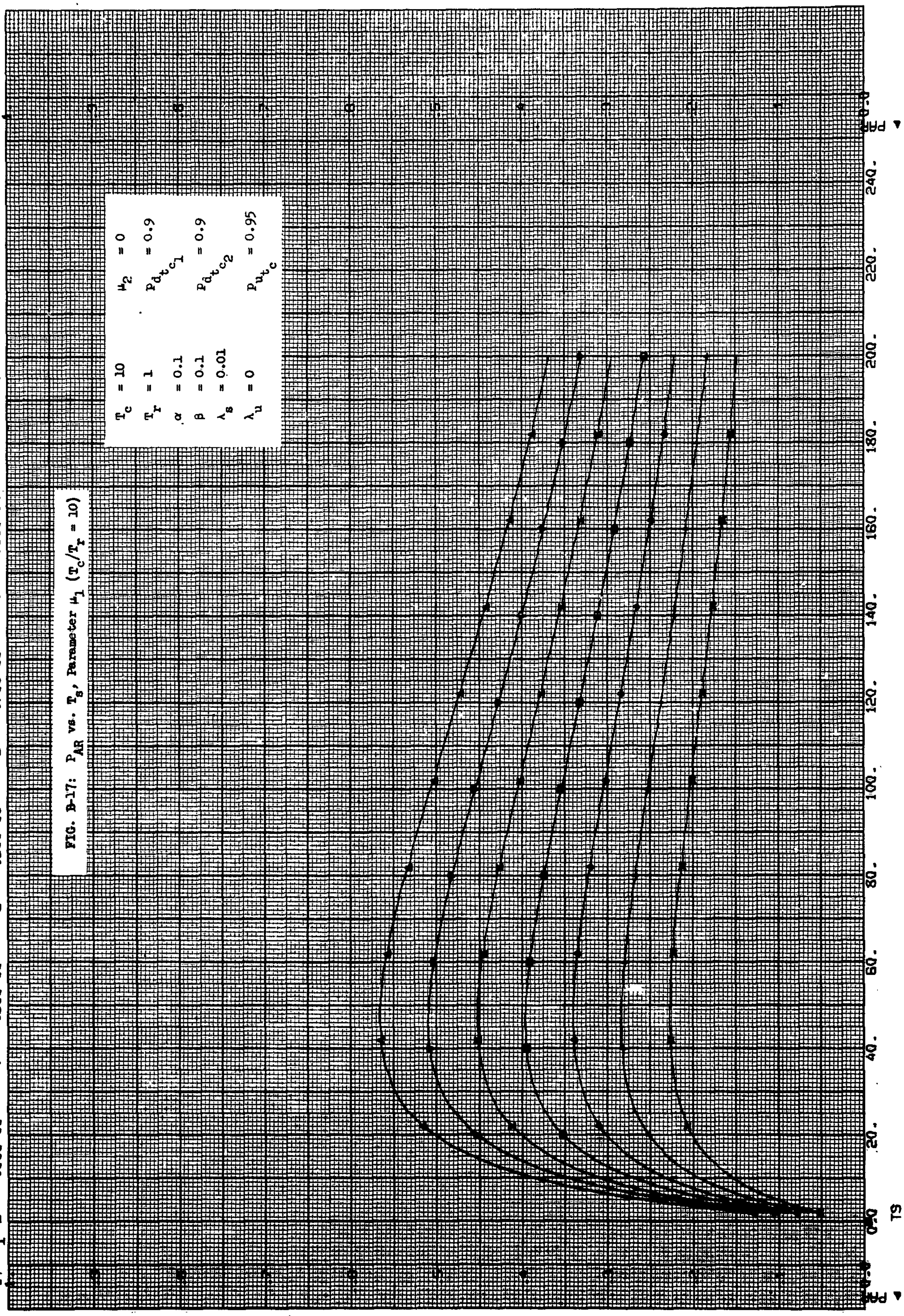

FIG. B-17: P_{AR} vs. T_s, Parameter μ_1 $(T_c/T_r = 10)$

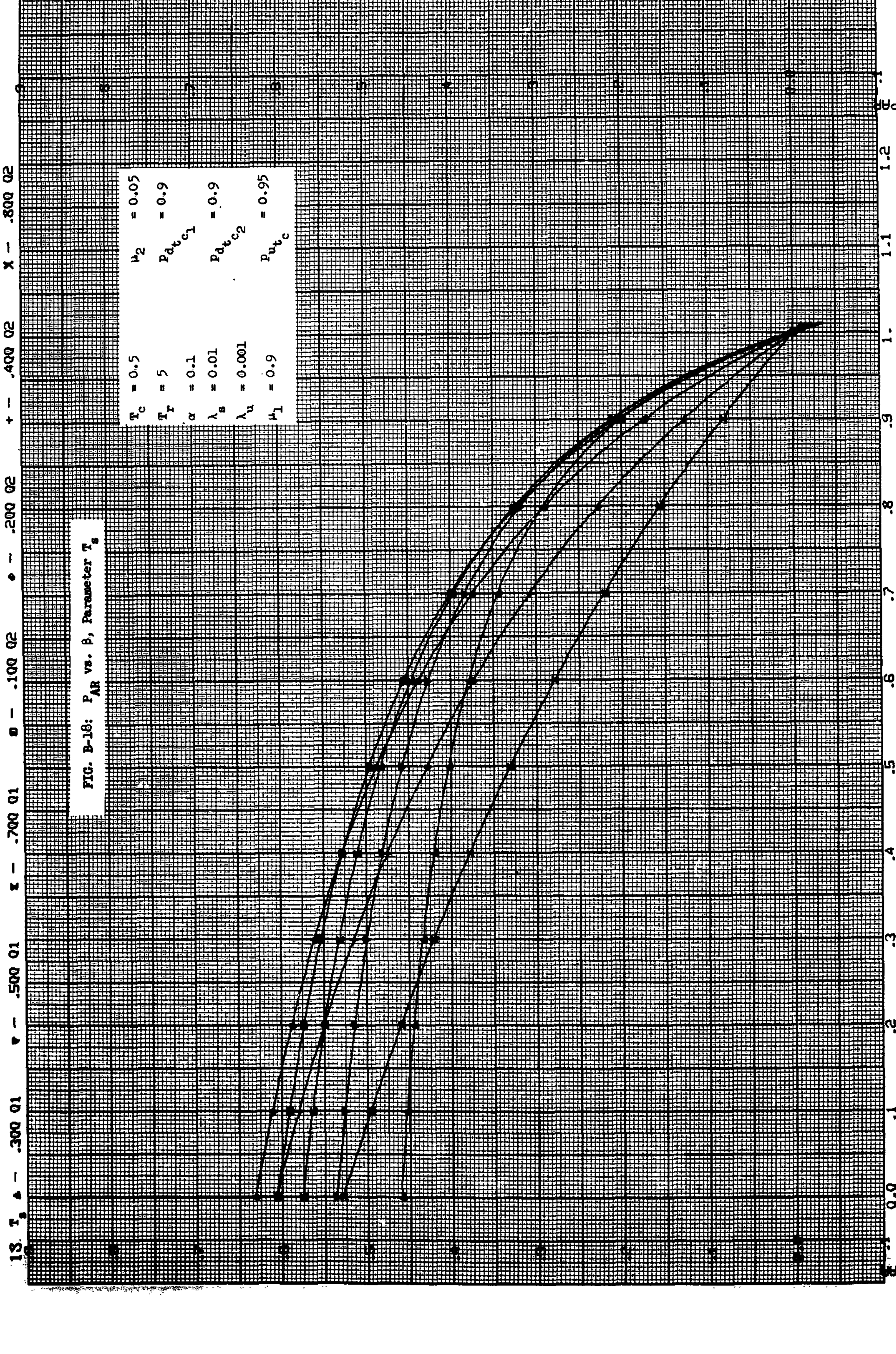

FIG. B-18: P_{AR} vs. β, Parameter T_s

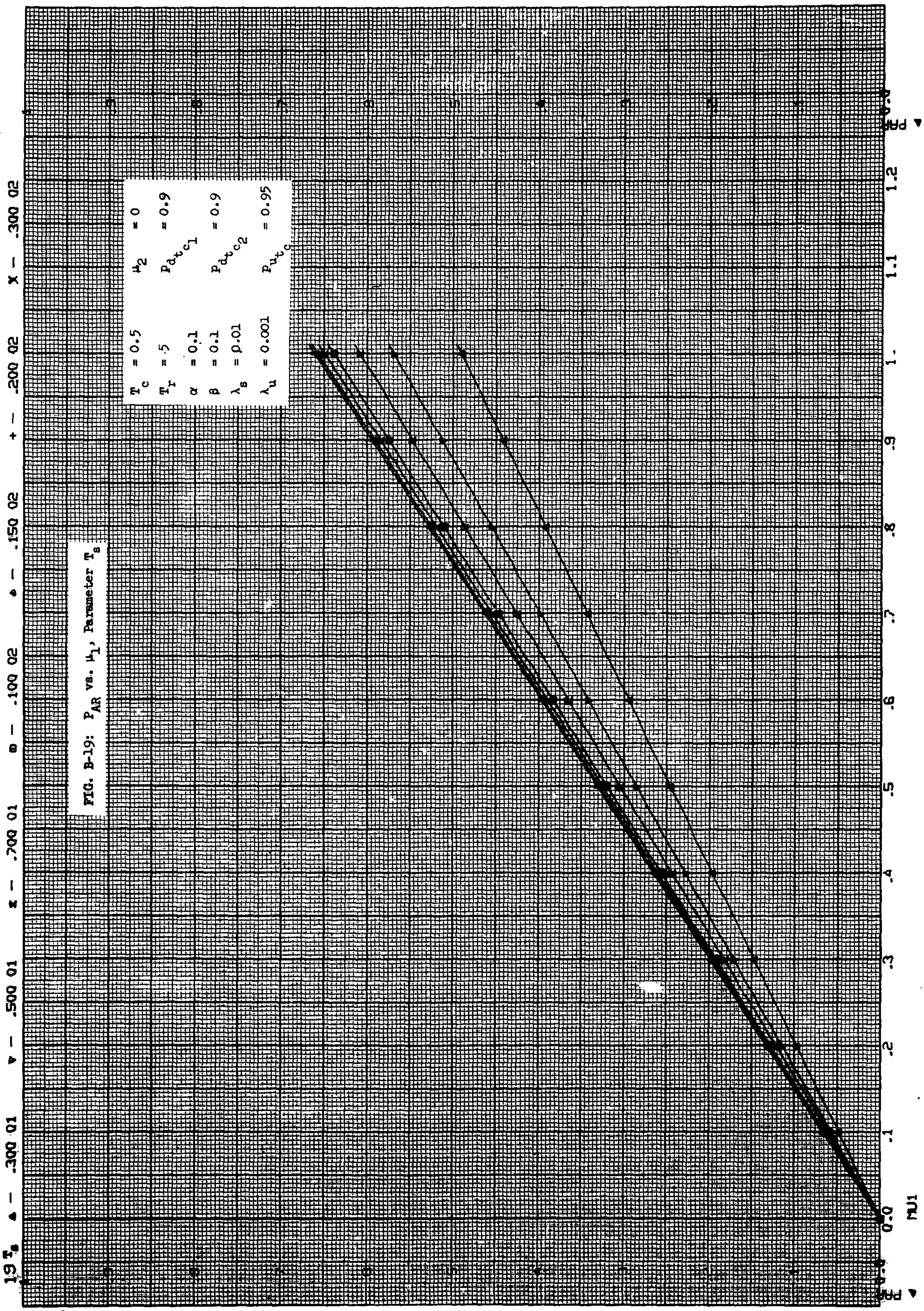

FIG. B-19: P_{AR} vs. μ_1, Parameter T_s

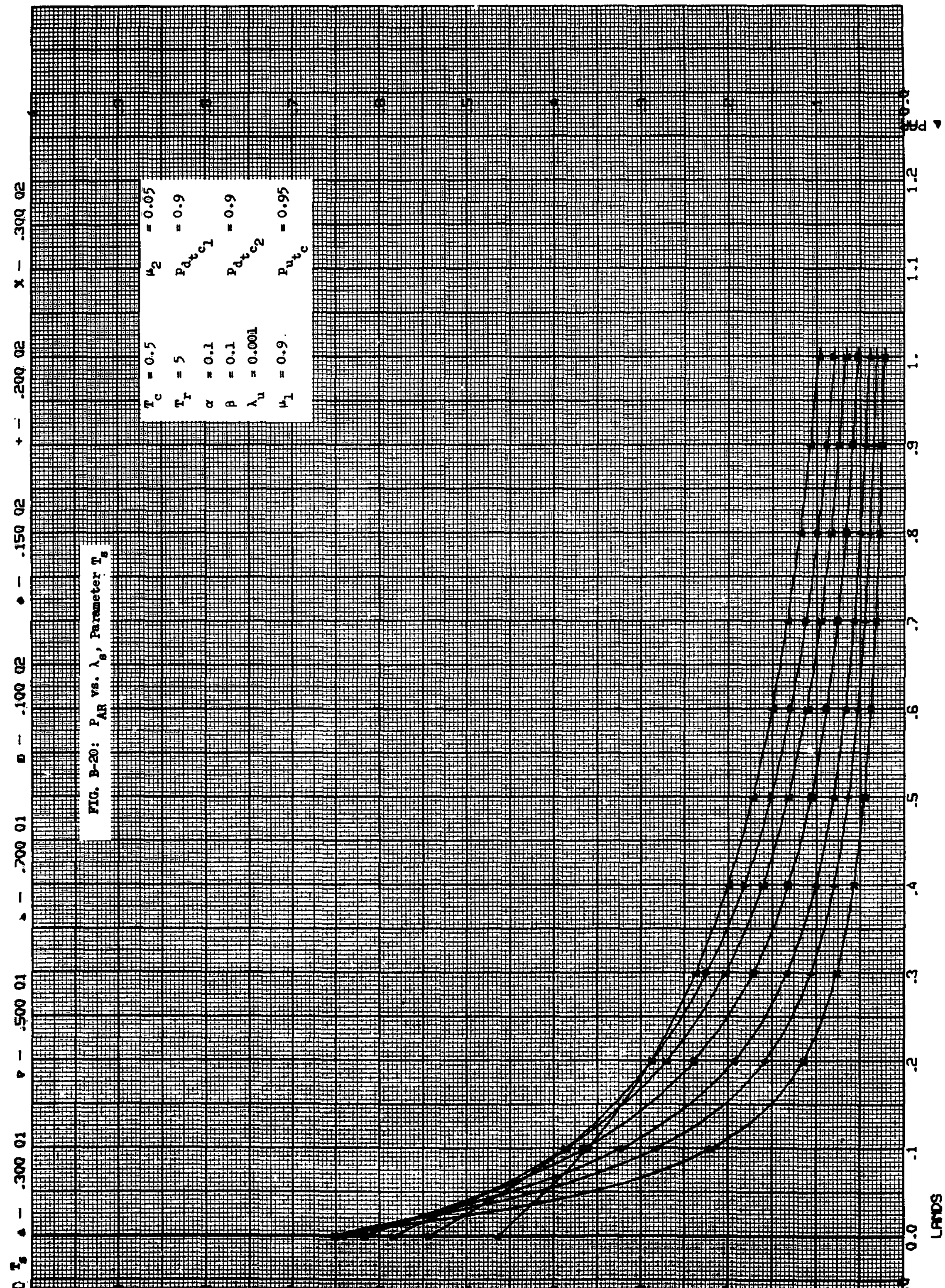

FIG. B-20: P_{AR} vs. λ_s, Parameter T_s

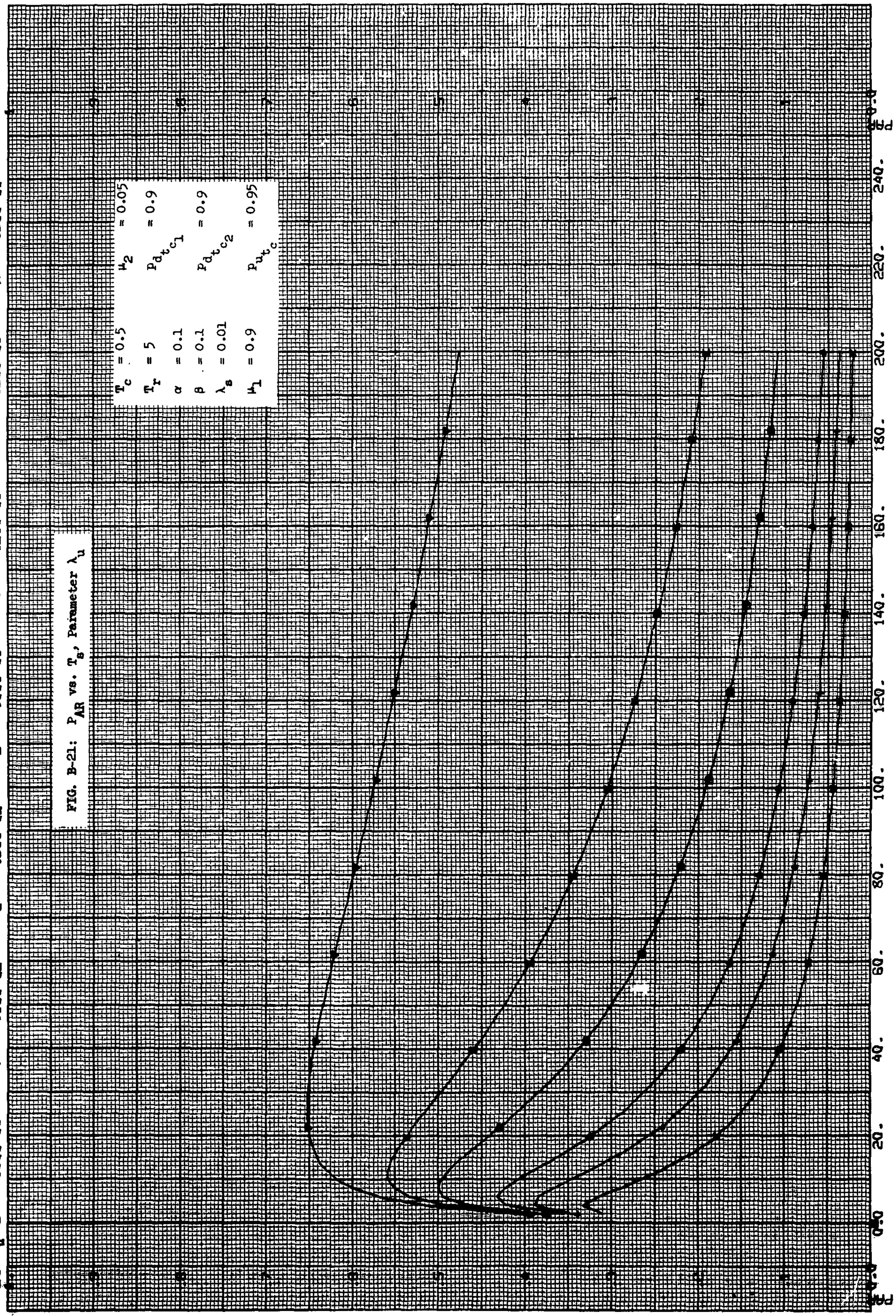

FIG. B-21: P_{AR} vs. T_s, Parameter λ_u

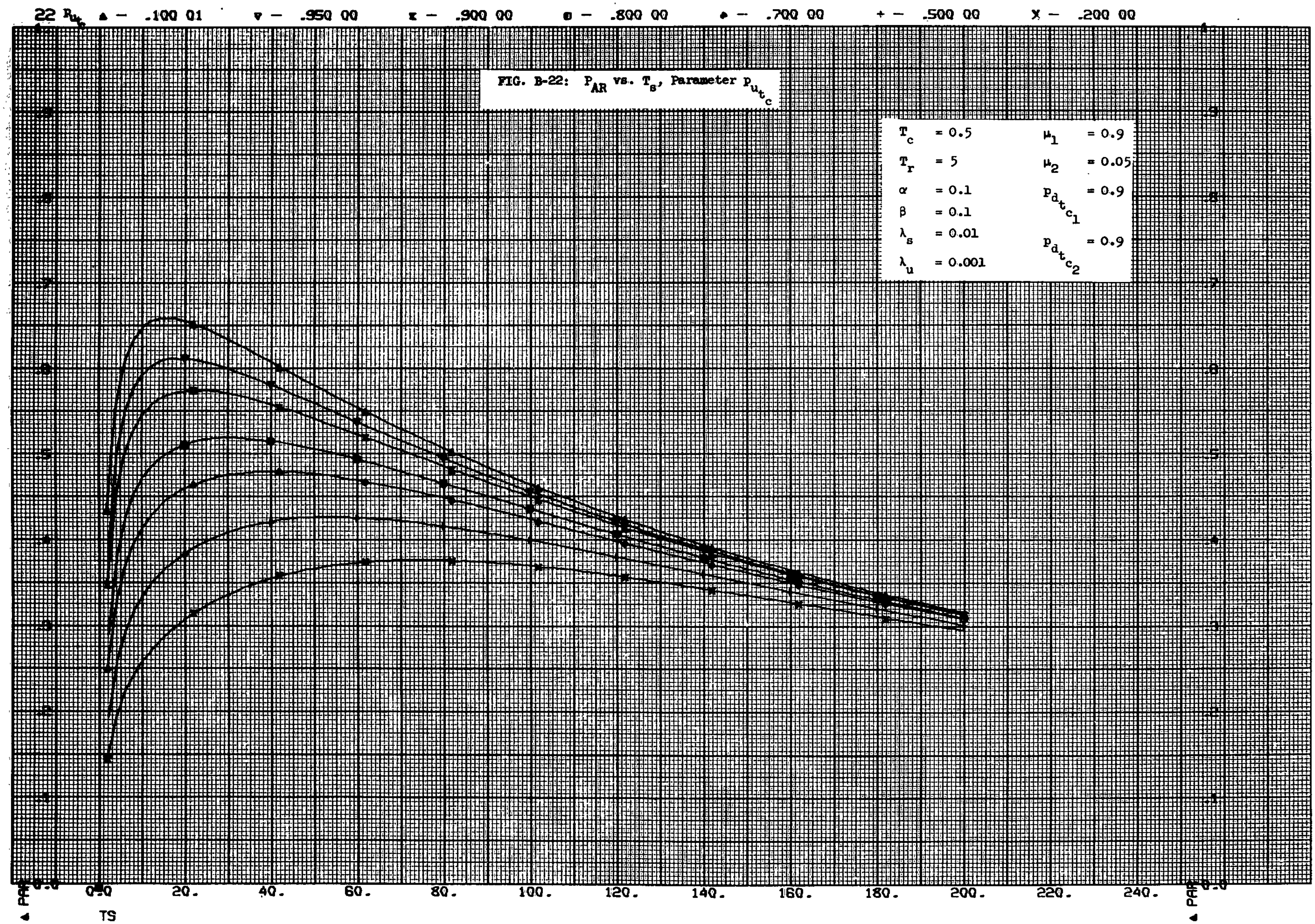

FIG. B-22: P_{AR} vs. T_s, Parameter $p_{u_{t_c}}$

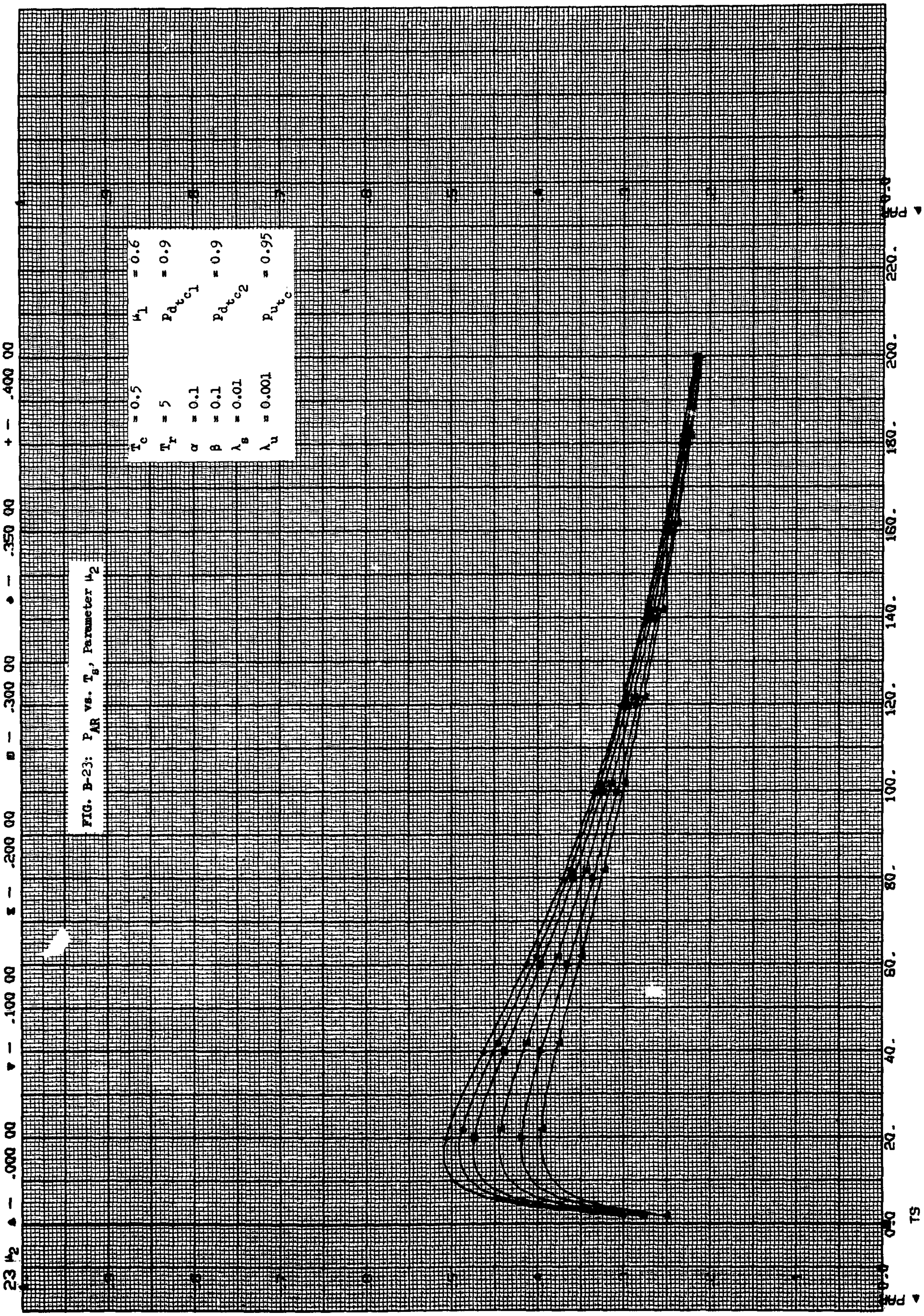

FIG. B-23: P_{AR} vs. T_s, Parameter μ_2

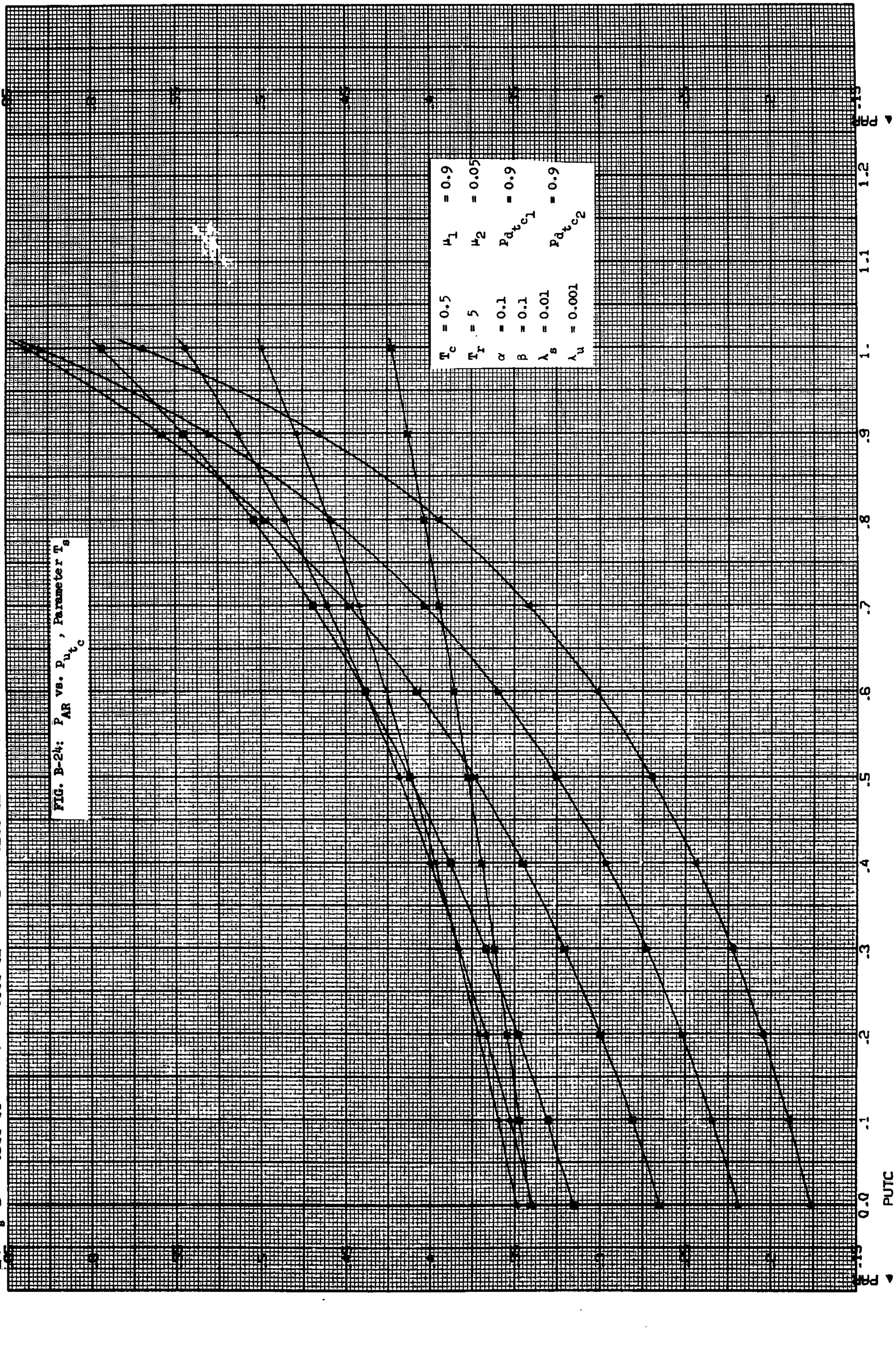

FIG. B-24: P_{AR} vs. $P_{u_{t_c}}$, Parameter T_s

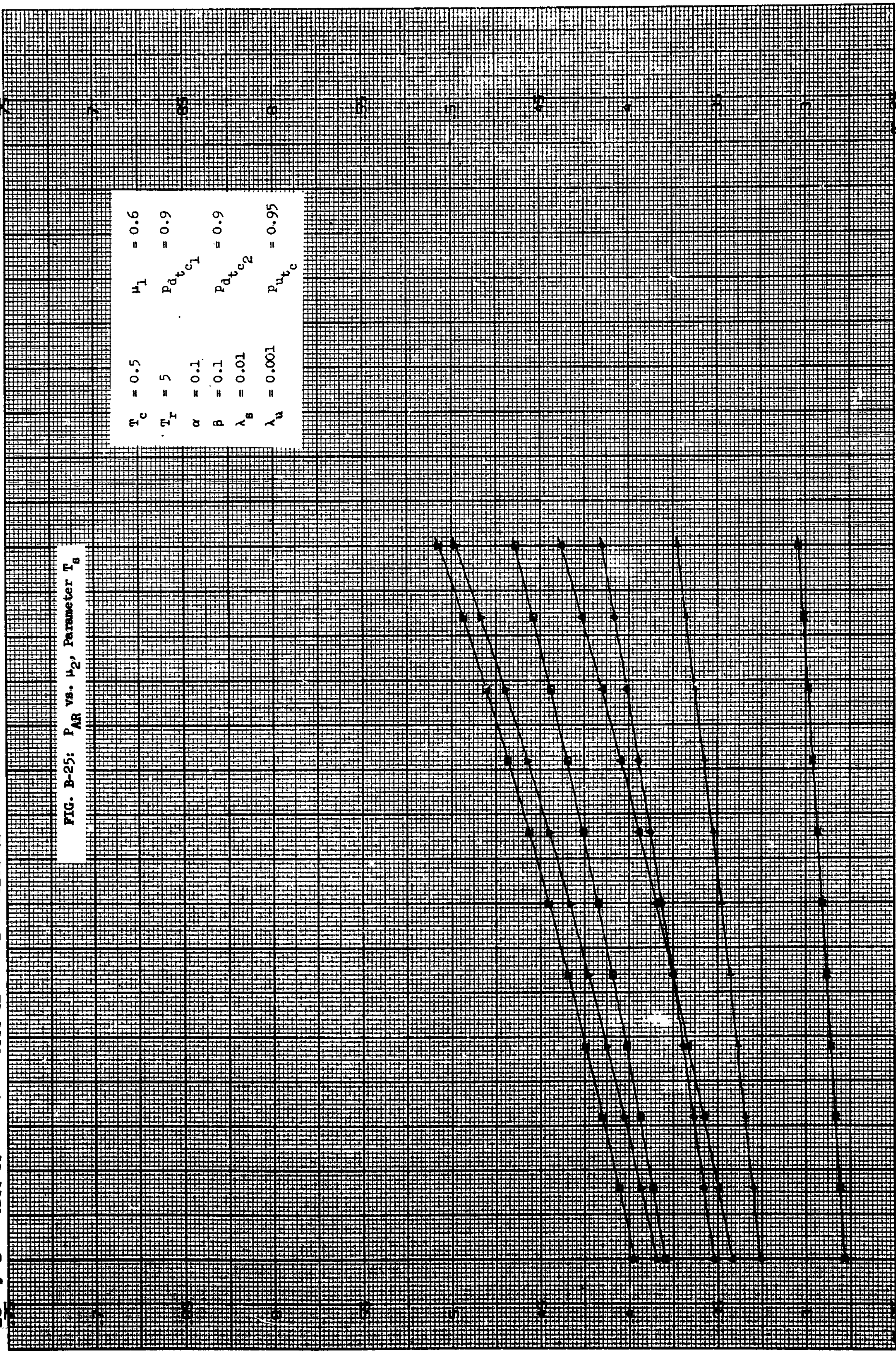

FIG. B-25: P_{AR} vs. μ_2, Parameter T_s

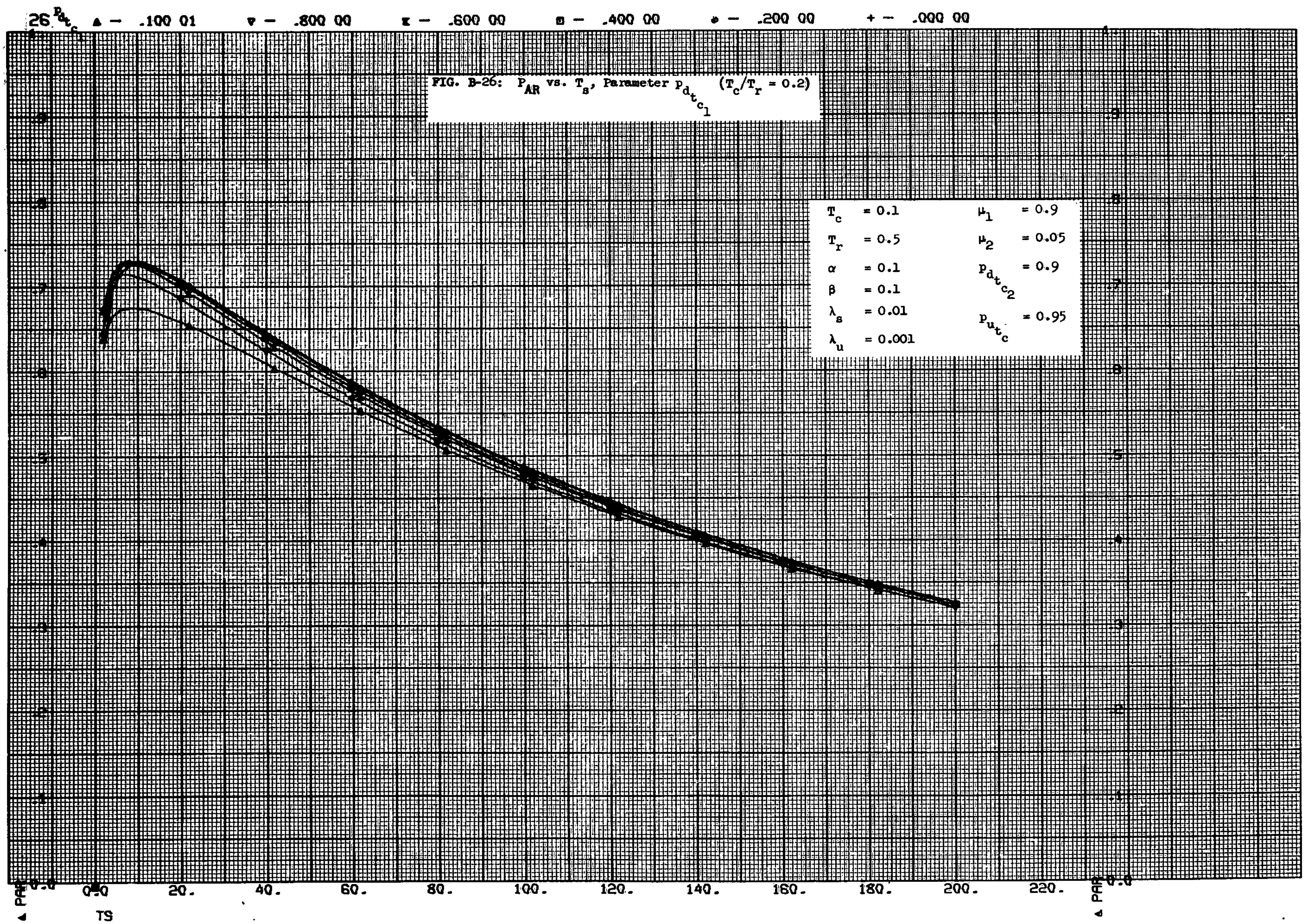

FIG. B-26: P_{AR} vs. T_s, Parameter $p_{d_{t_{c_1}}}$ (T_c/T_r = 0.2)

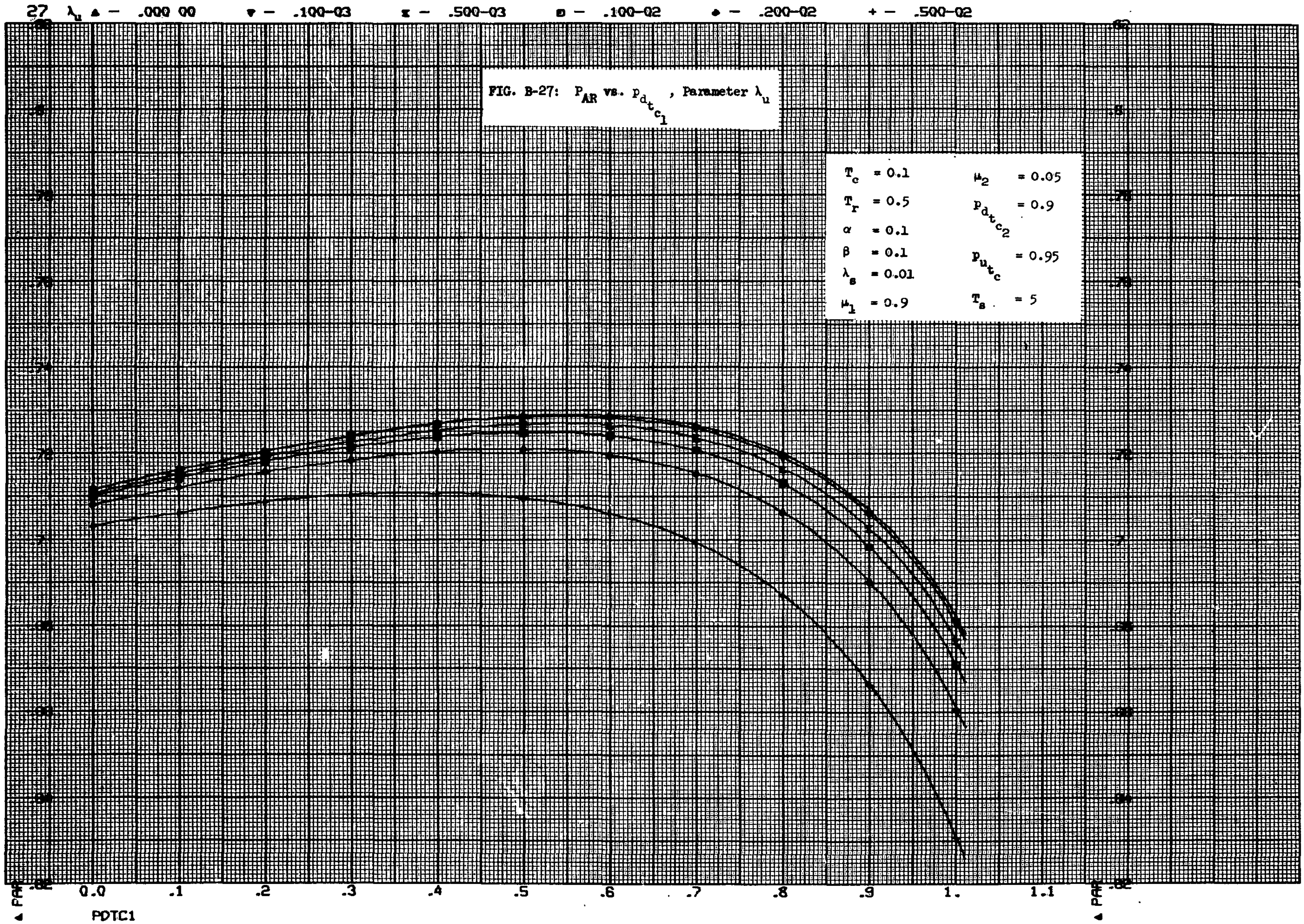

FIG. B-27: P_{AR} vs. $p_{d_{t_{c_1}}}$, Parameter λ_u

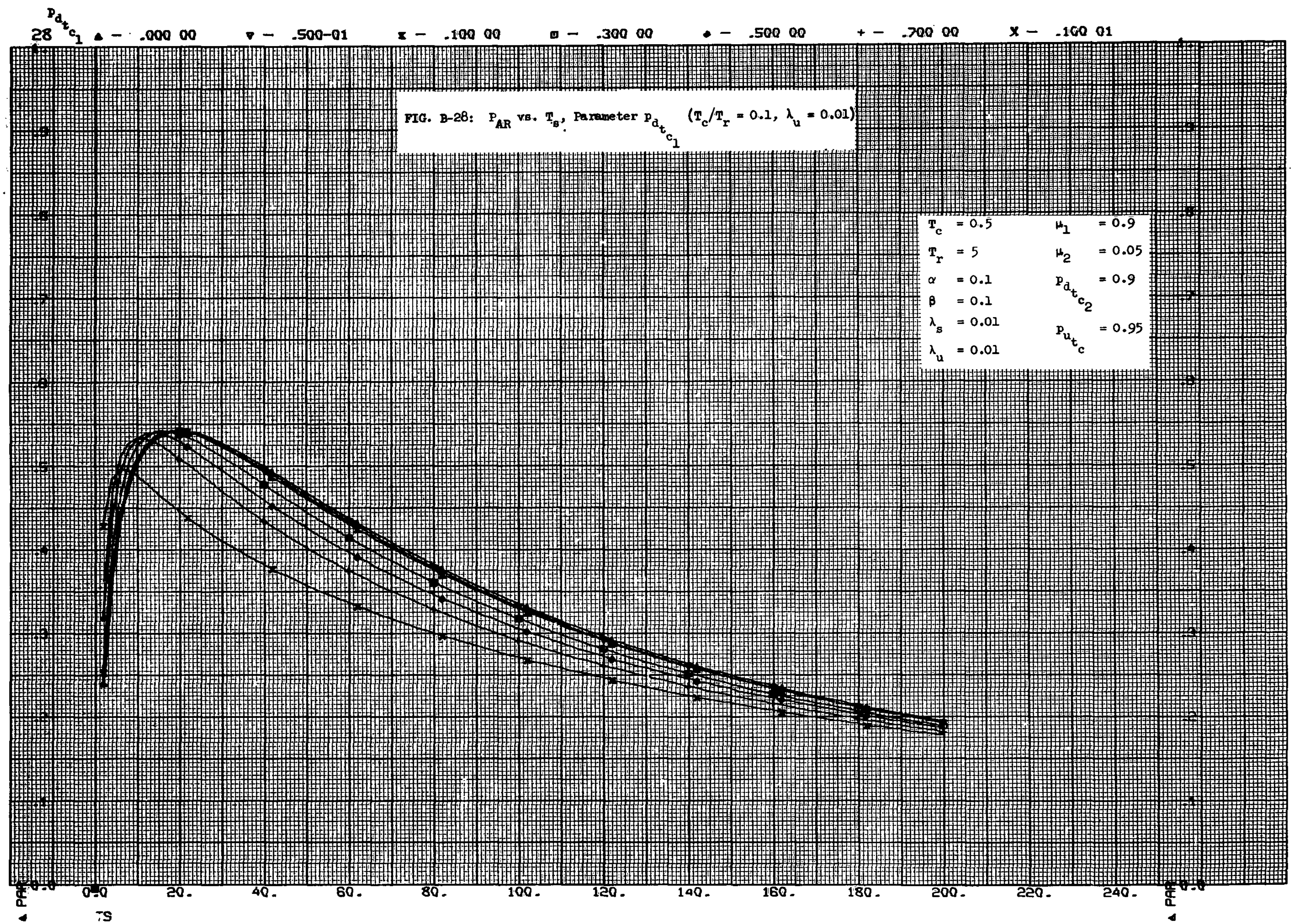

FIG. B-28: P_{AR} vs. T_s, Parameter $p_{d_{t_{c_1}}}$ $(T_c/T_r = 0.1,\ \lambda_u = 0.01)$

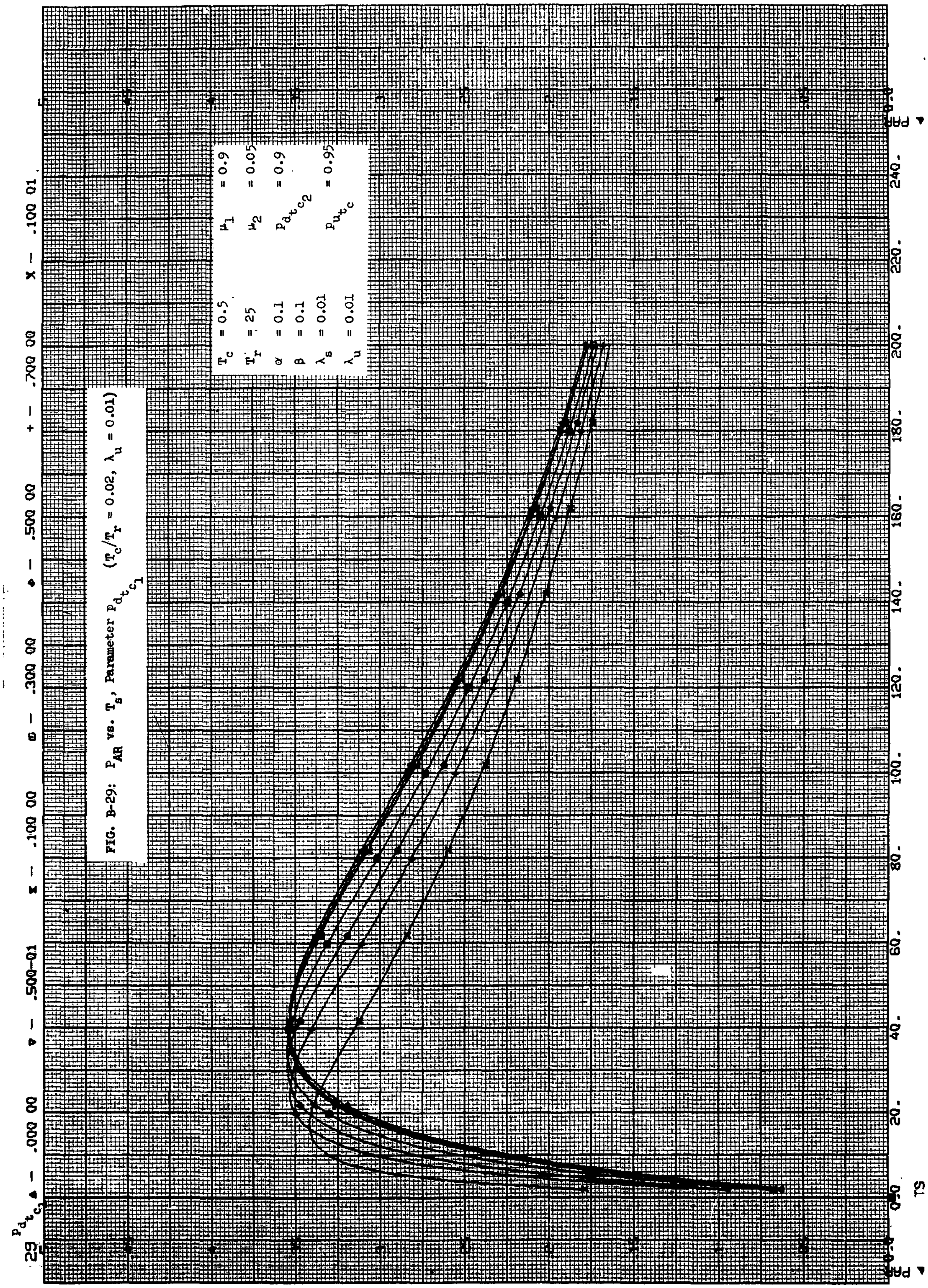

FIG. B-29: P_{AR} vs. T_s, Parameter $p_{d_{t_{c_1}}}$ $(T_c/T_r = 0.02,\ \lambda_u = 0.01)$

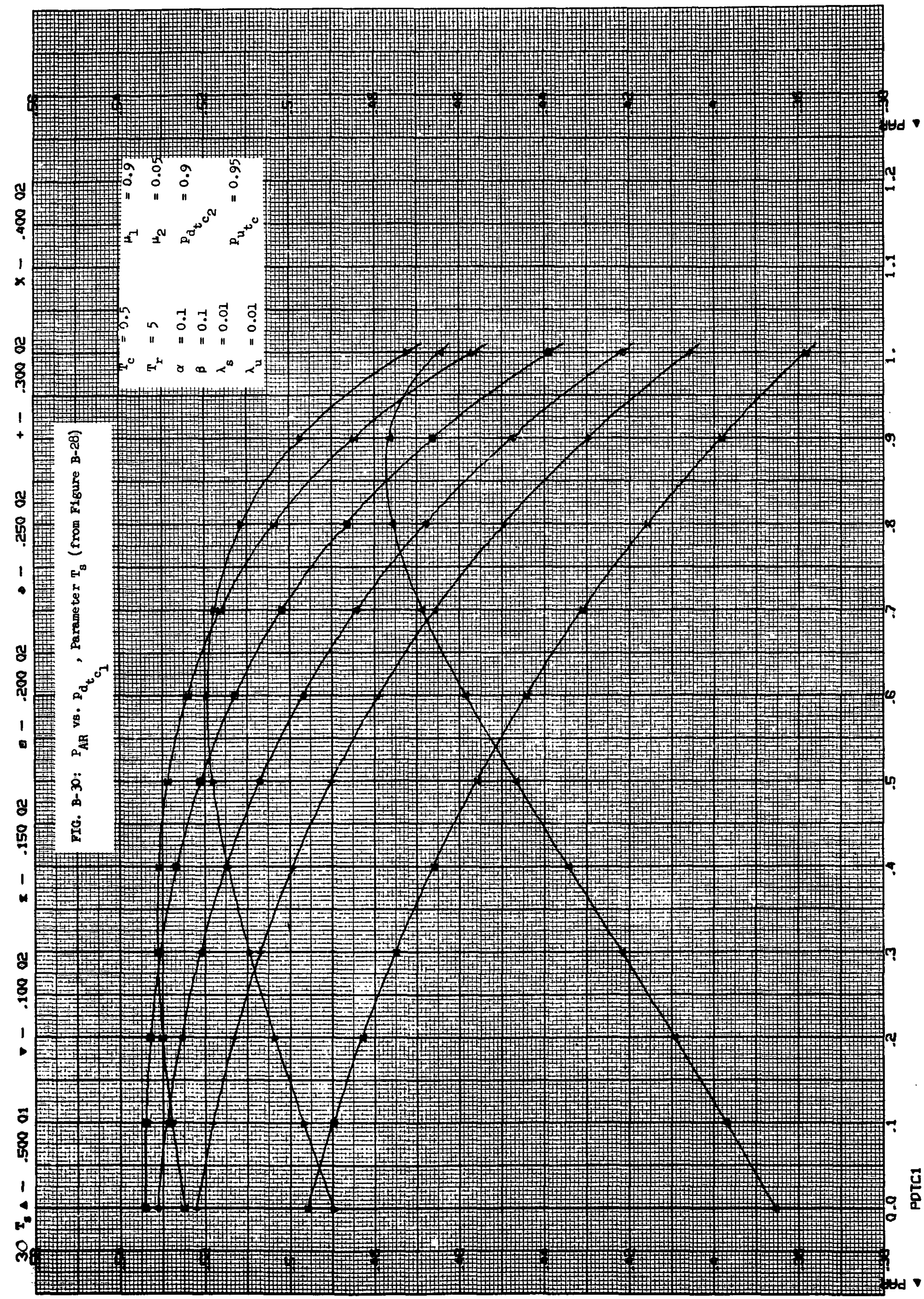

FIG. B-30: P_{AR} vs. $P_{d_{t_{c_1}}}$, Parameter T_s (from Figure B-28)

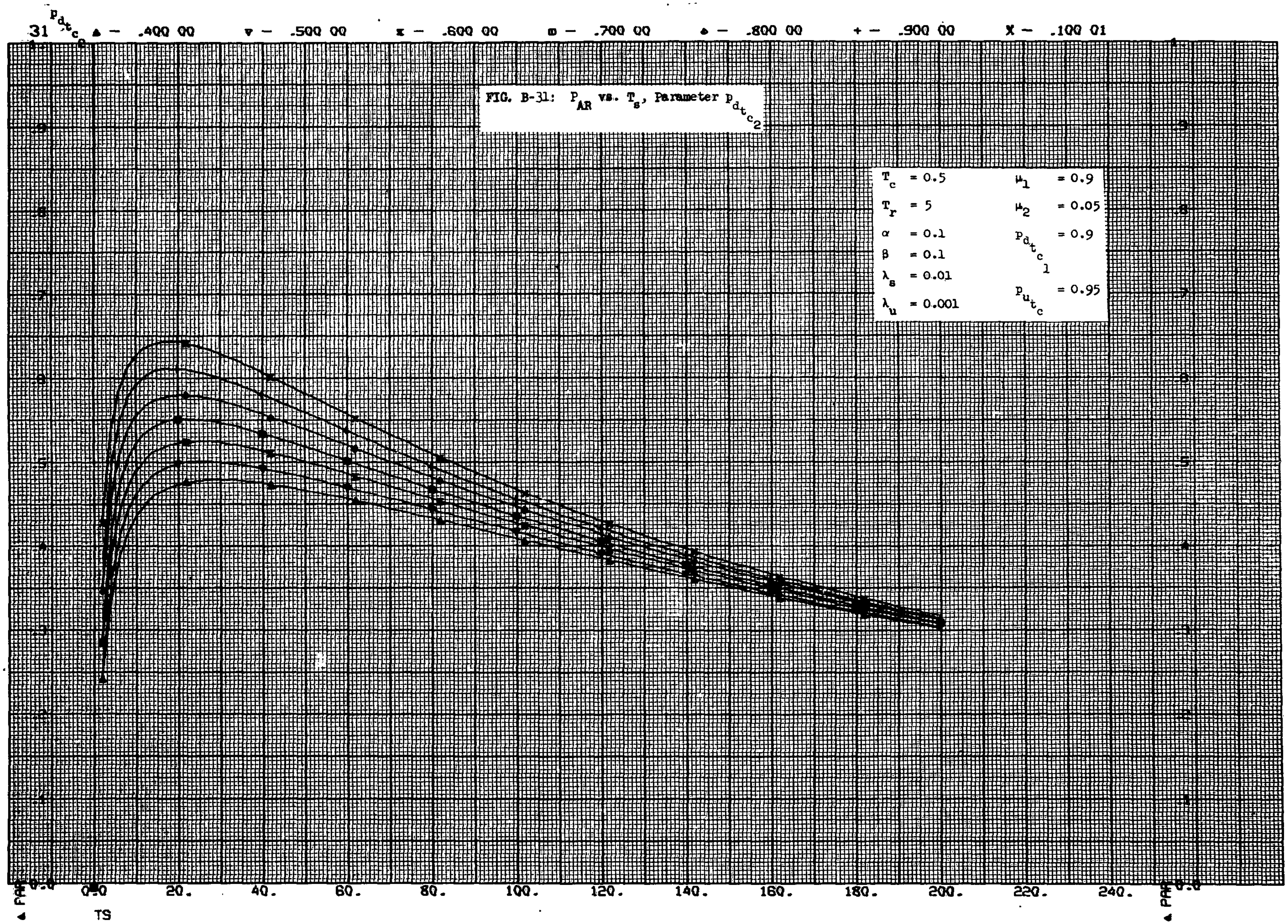

FIG. B-31: P_{AR} vs. T_s, Parameter $p_{d_{t_{c_2}}}$

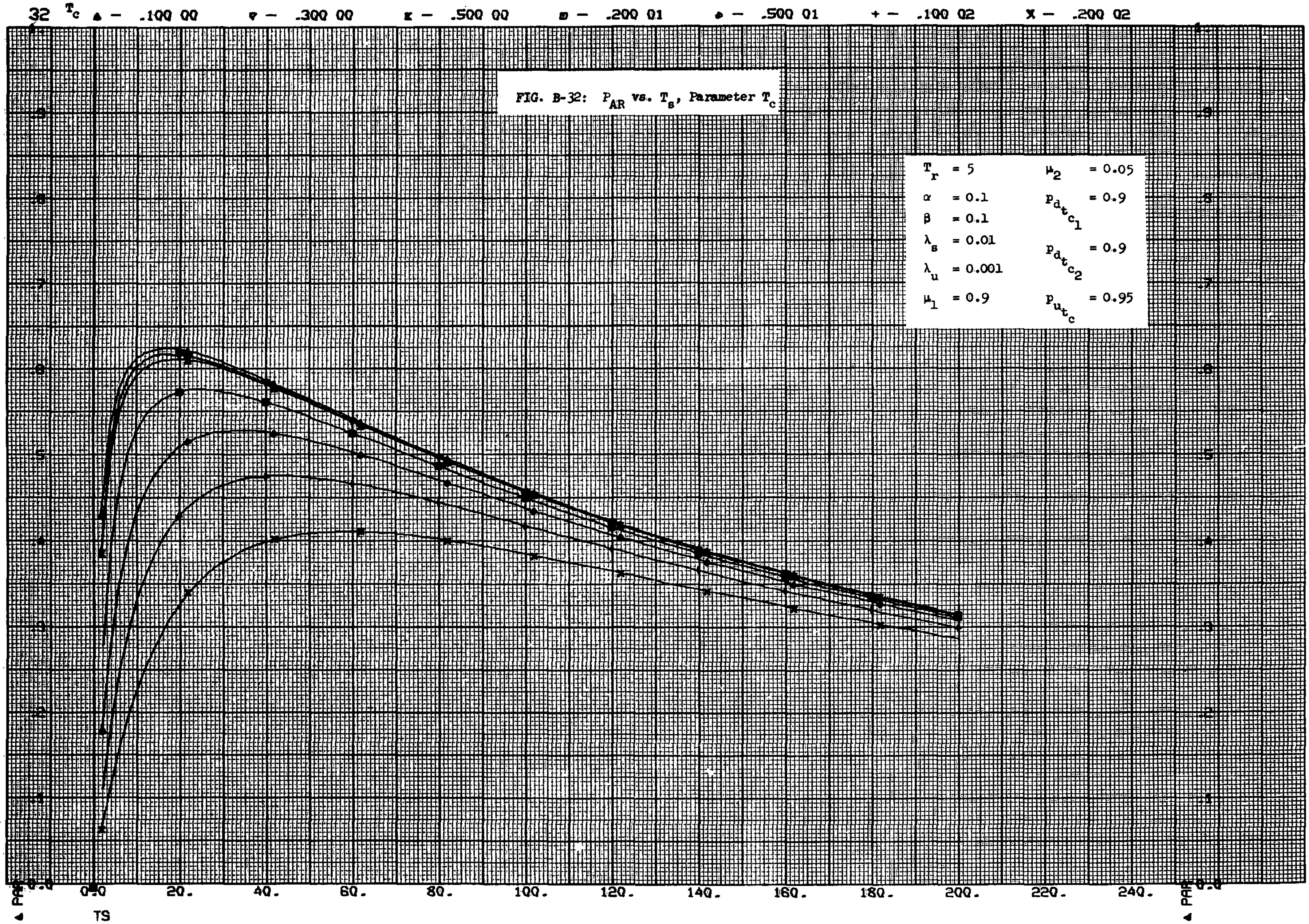

FIG. B-32: P_{AR} vs. T_s, Parameter T_c

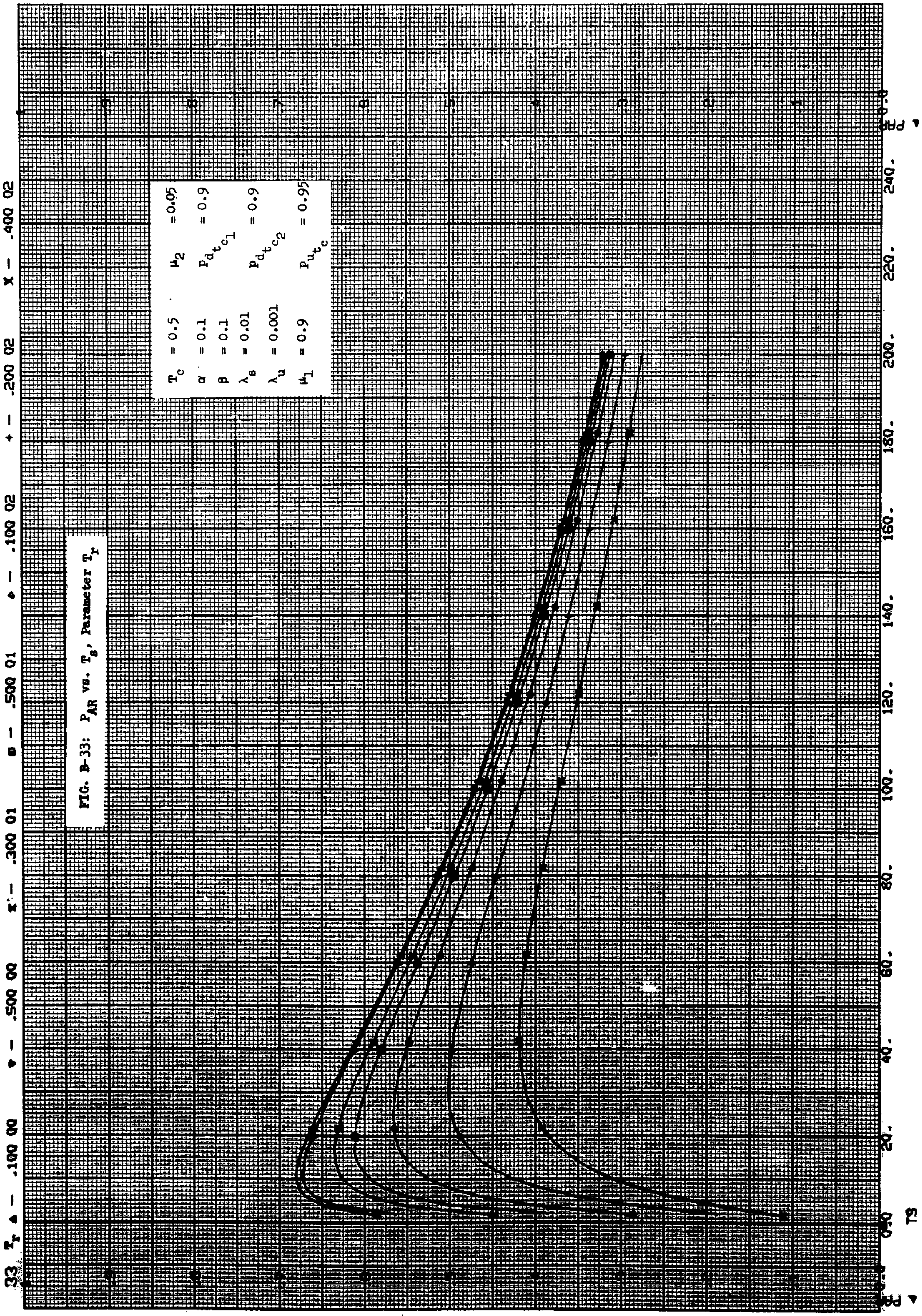

FIG. B-33: P_{AR} vs. T_s, Parameter T_r

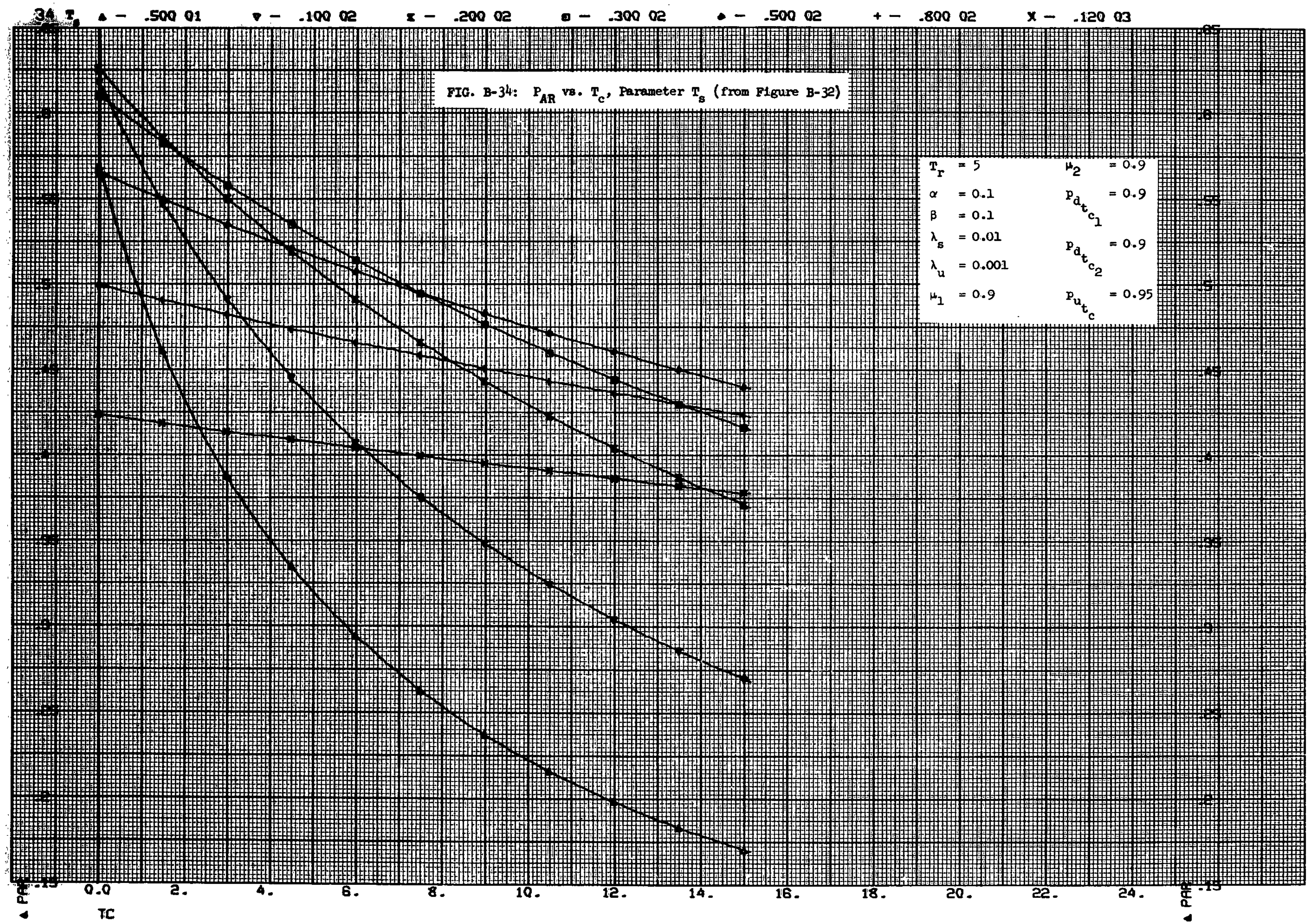

FIG. B-34: P_{AR} vs. T_c, Parameter T_s (from Figure B-32)

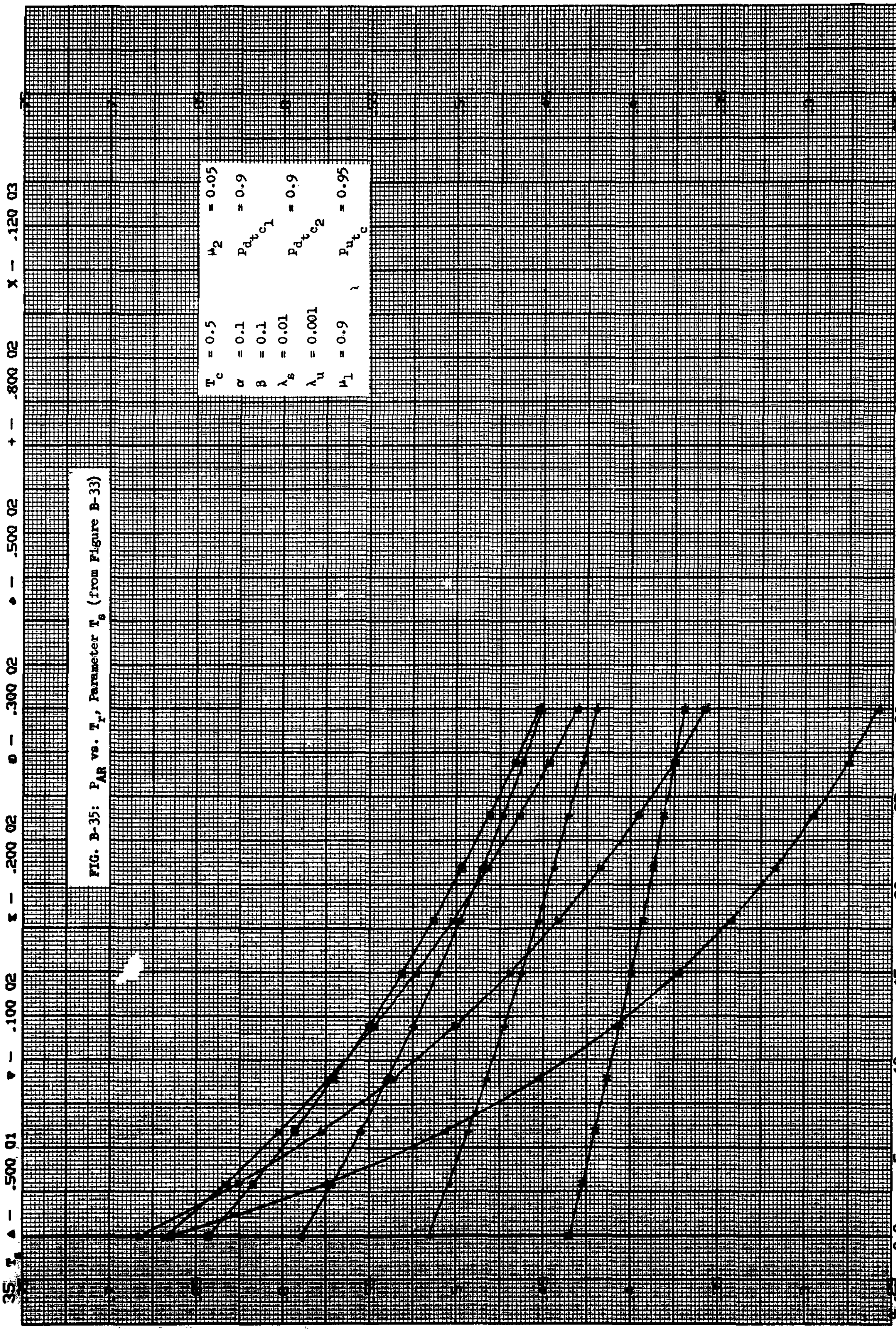

FIG. B-35: P_{AR} vs. T_r, Parameter T_s (from Figure B-33)

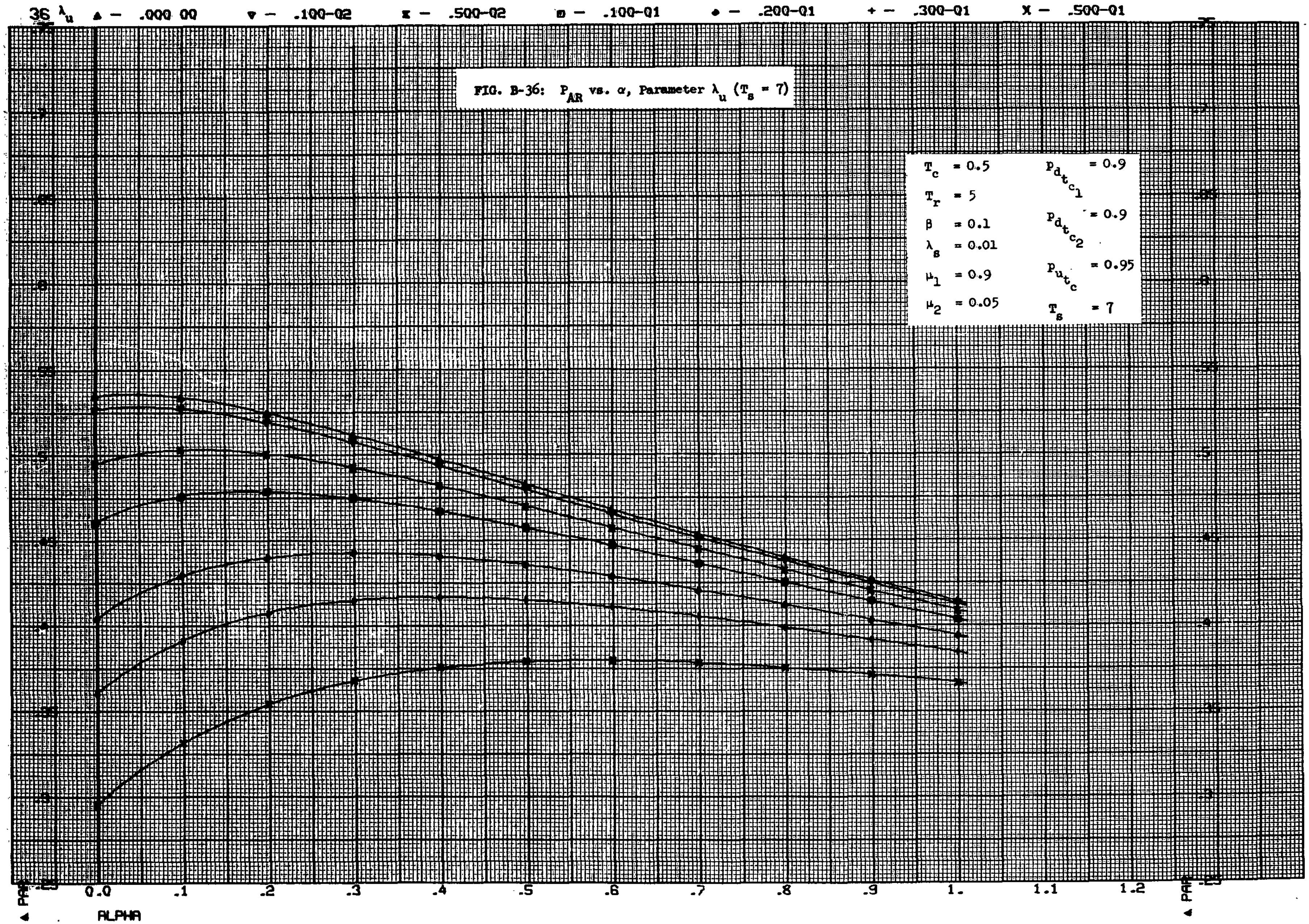

FIG. B-36: P_{AR} vs. α, Parameter λ_u (T_s = 7)

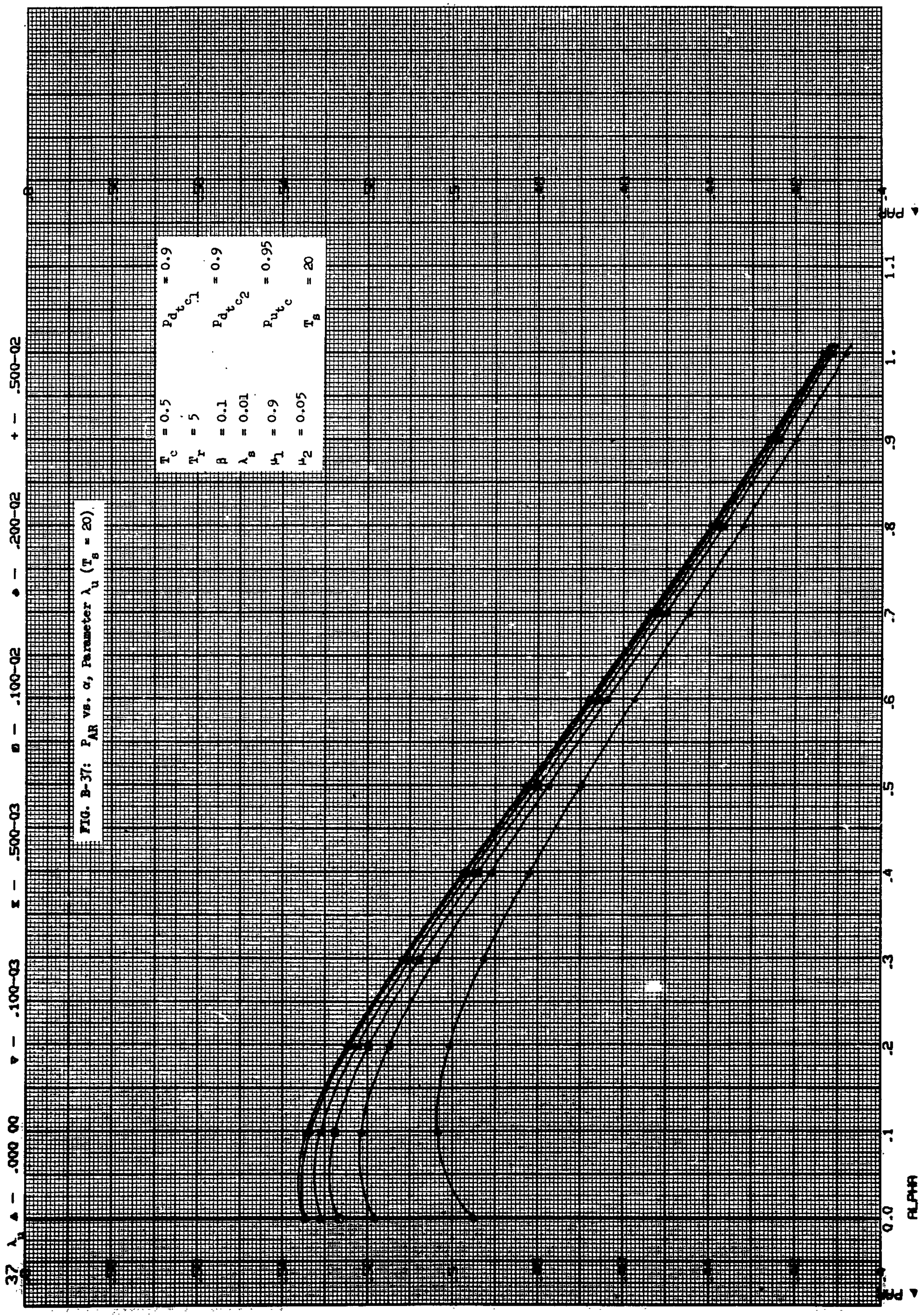

FIG. B-37: P_{AR} vs. α, Parameter λ_u (T_s = 20)

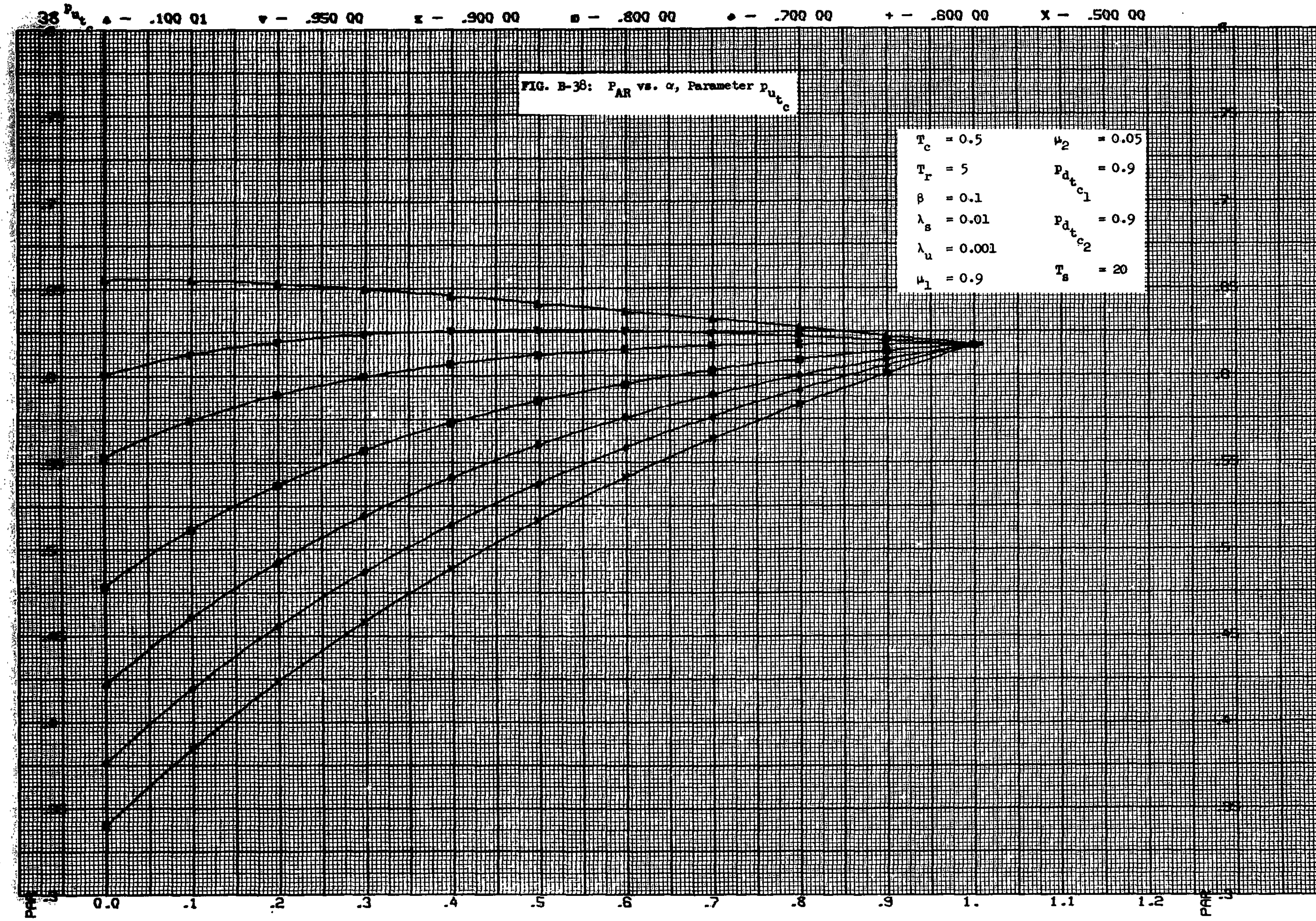

FIG. B-38: P_{AR} vs. α, Parameter $p_{u_{t_c}}$

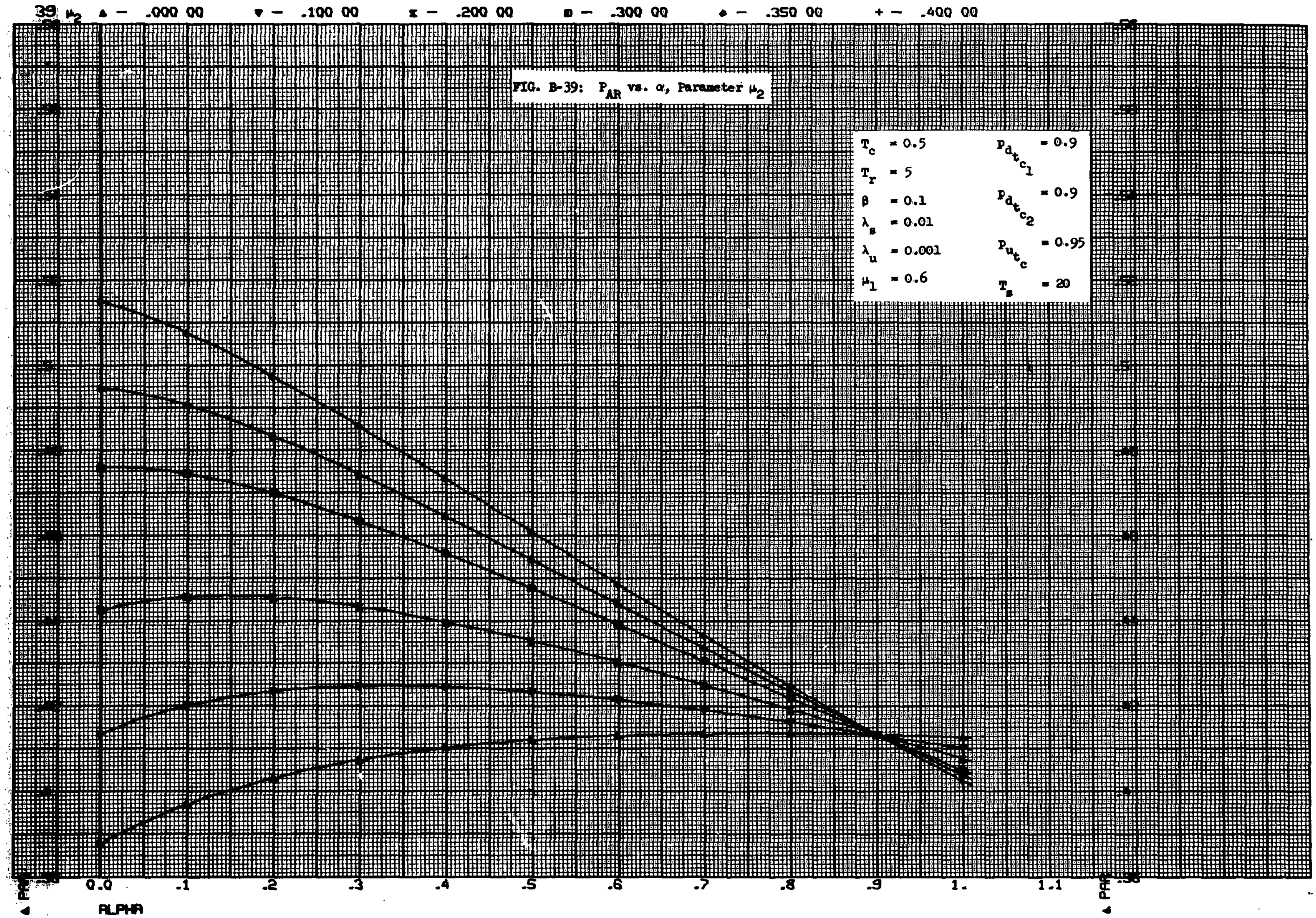

FIG. B-39: P_{AR} vs. α, Parameter μ_2

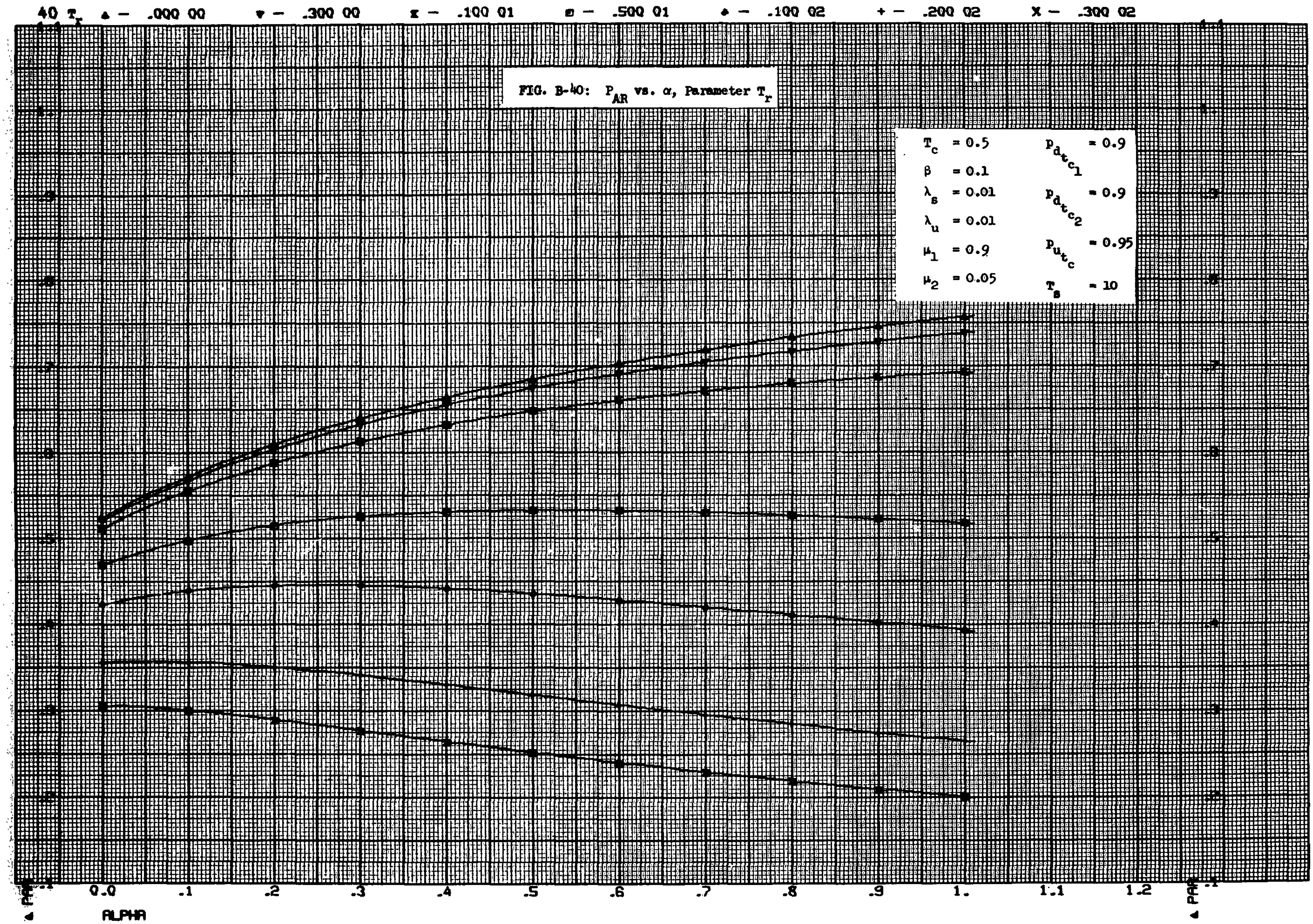

FIG. B-40: P_{AR} vs. α, Parameter T_r

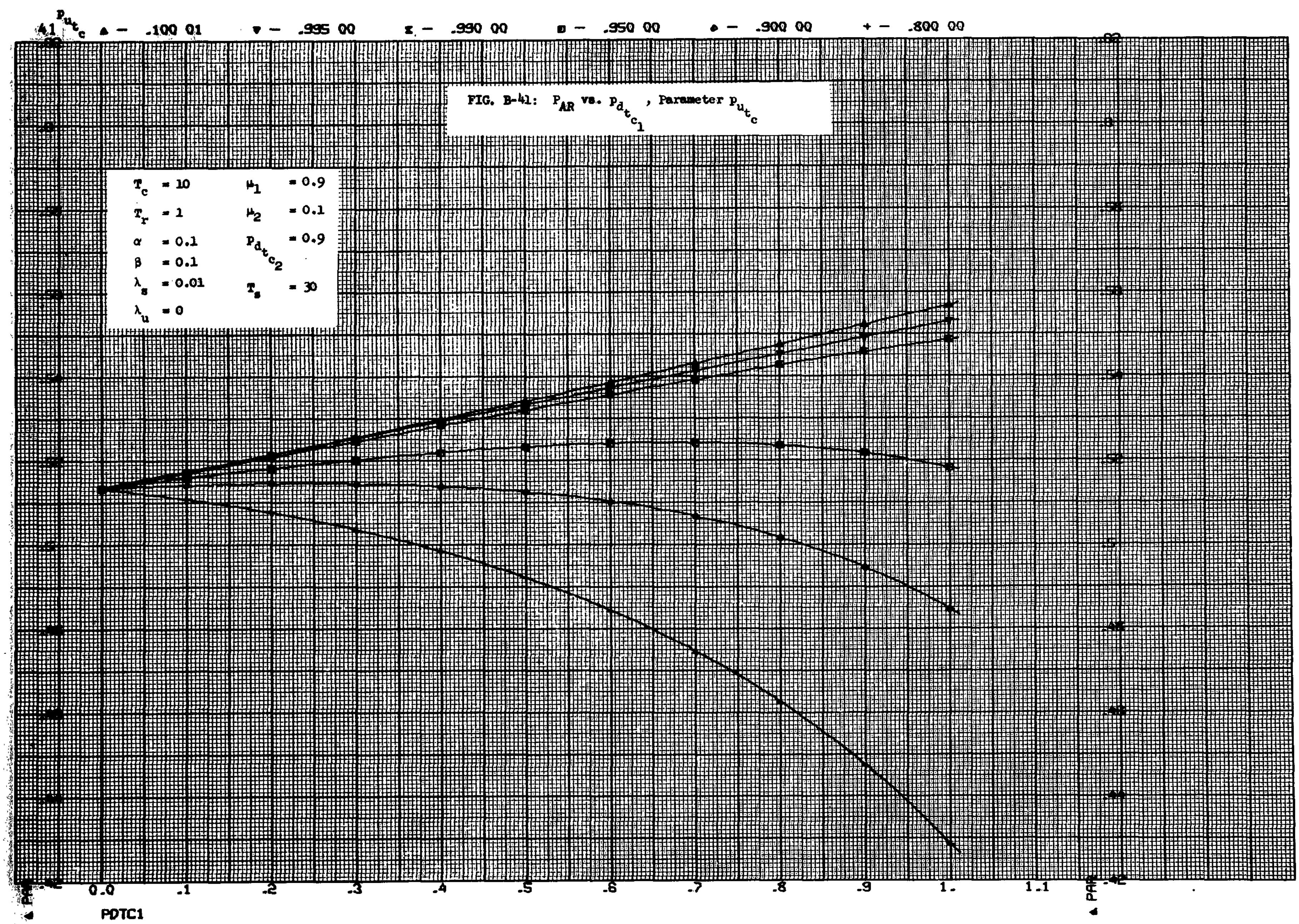

FIG. B-41: P_{AR} vs. $p_{d_{t_{c_1}}}$, Parameter $p_{u_{t_c}}$

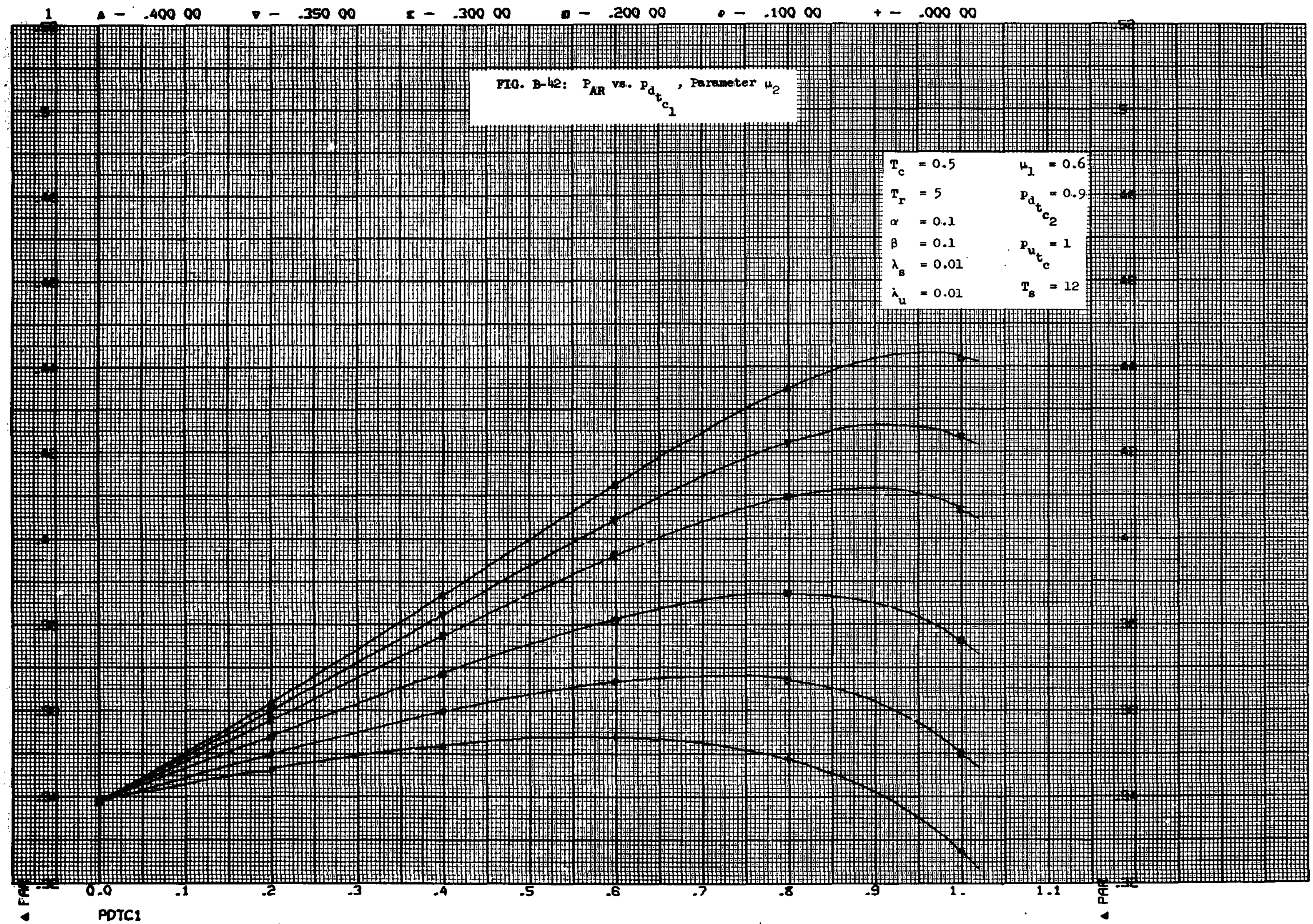

FIG. B-42: P_{AR} vs. $p_{d_{t_{c_1}}}$, Parameter μ_2

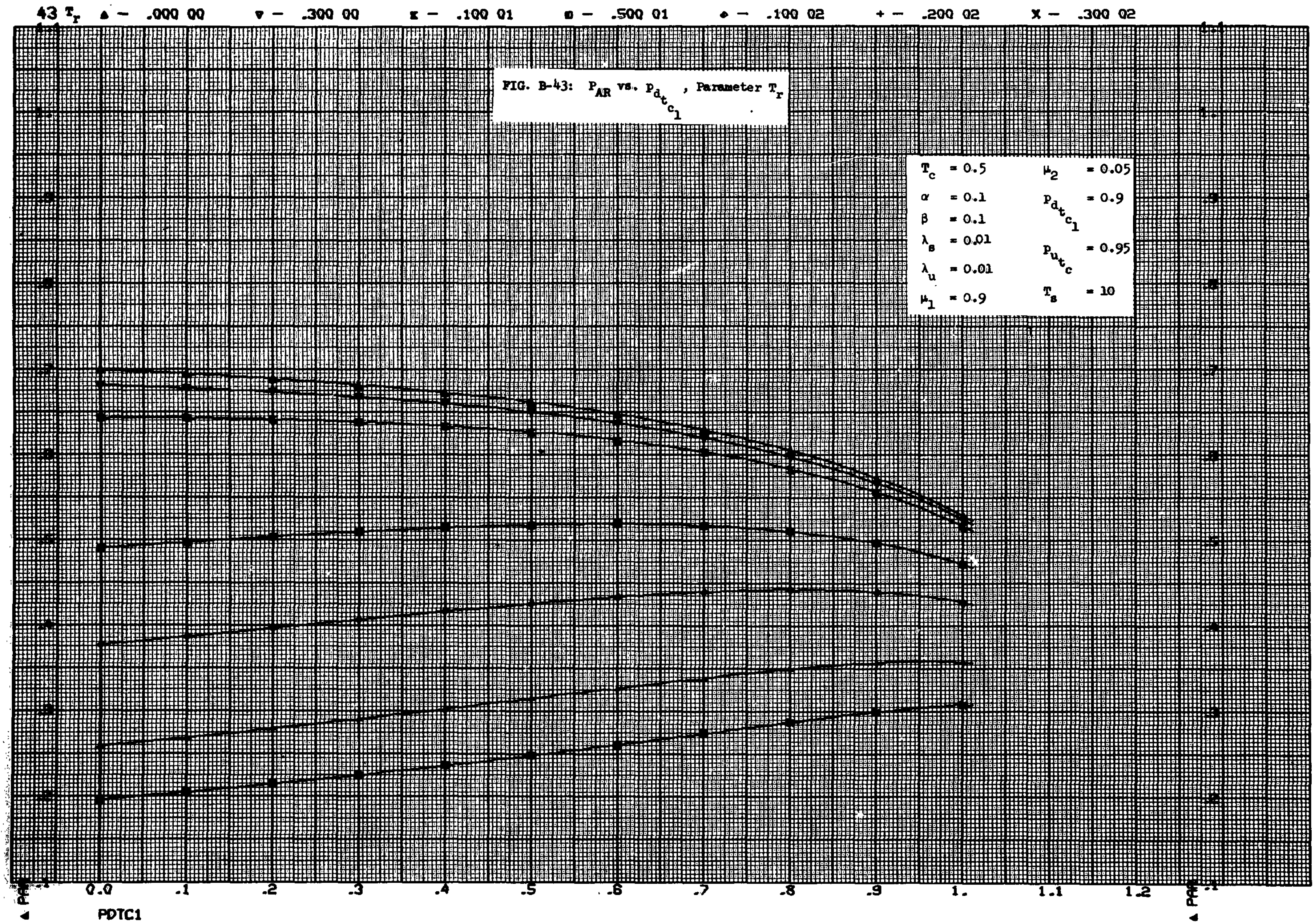

FIG. B-43: P_{AR} vs. $p_{d_{t_{c_1}}}$, Parameter T_r

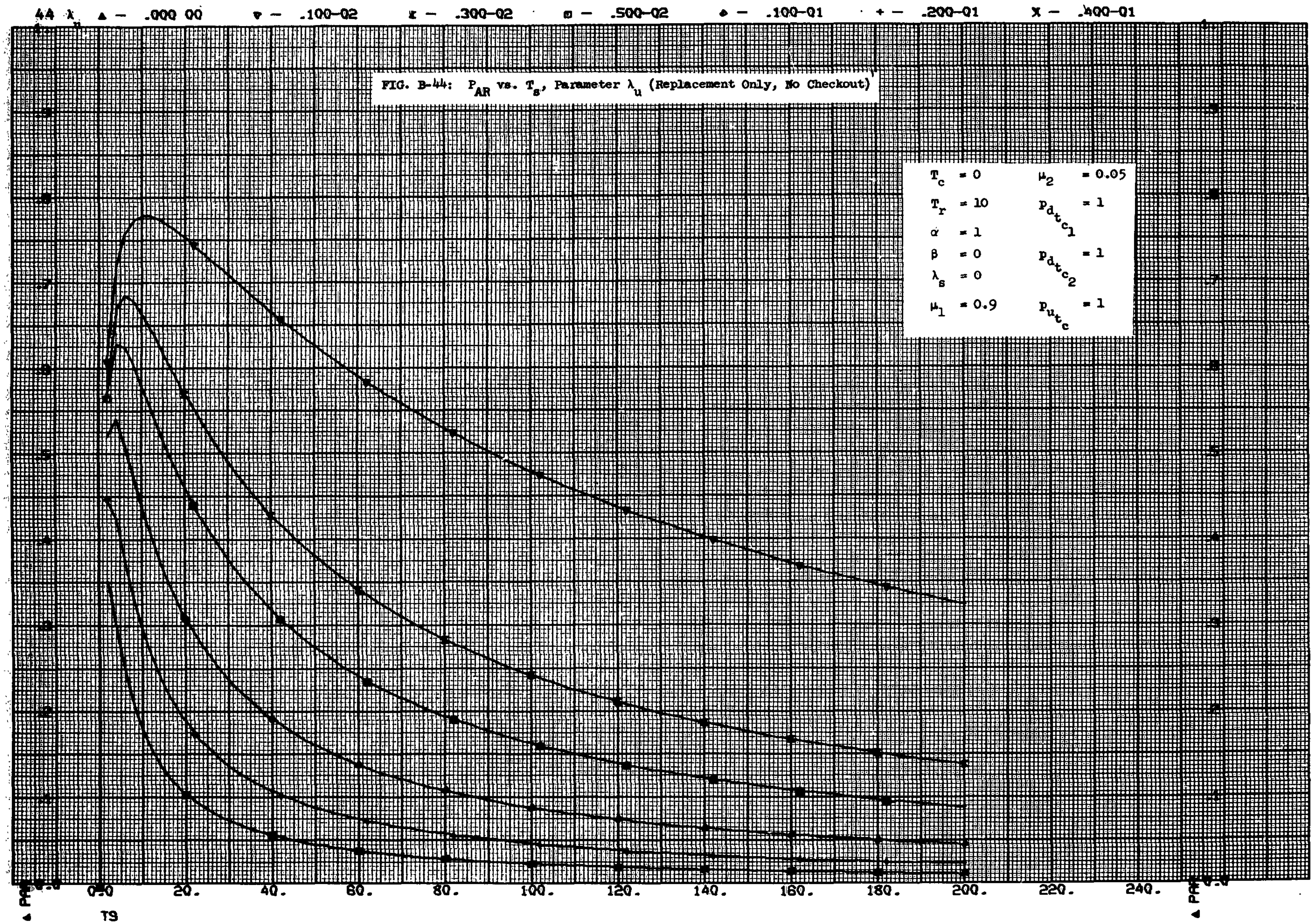

FIG. B-44: P_{AR} vs. T_s, Parameter λ_u (Replacement Only, No Checkout)

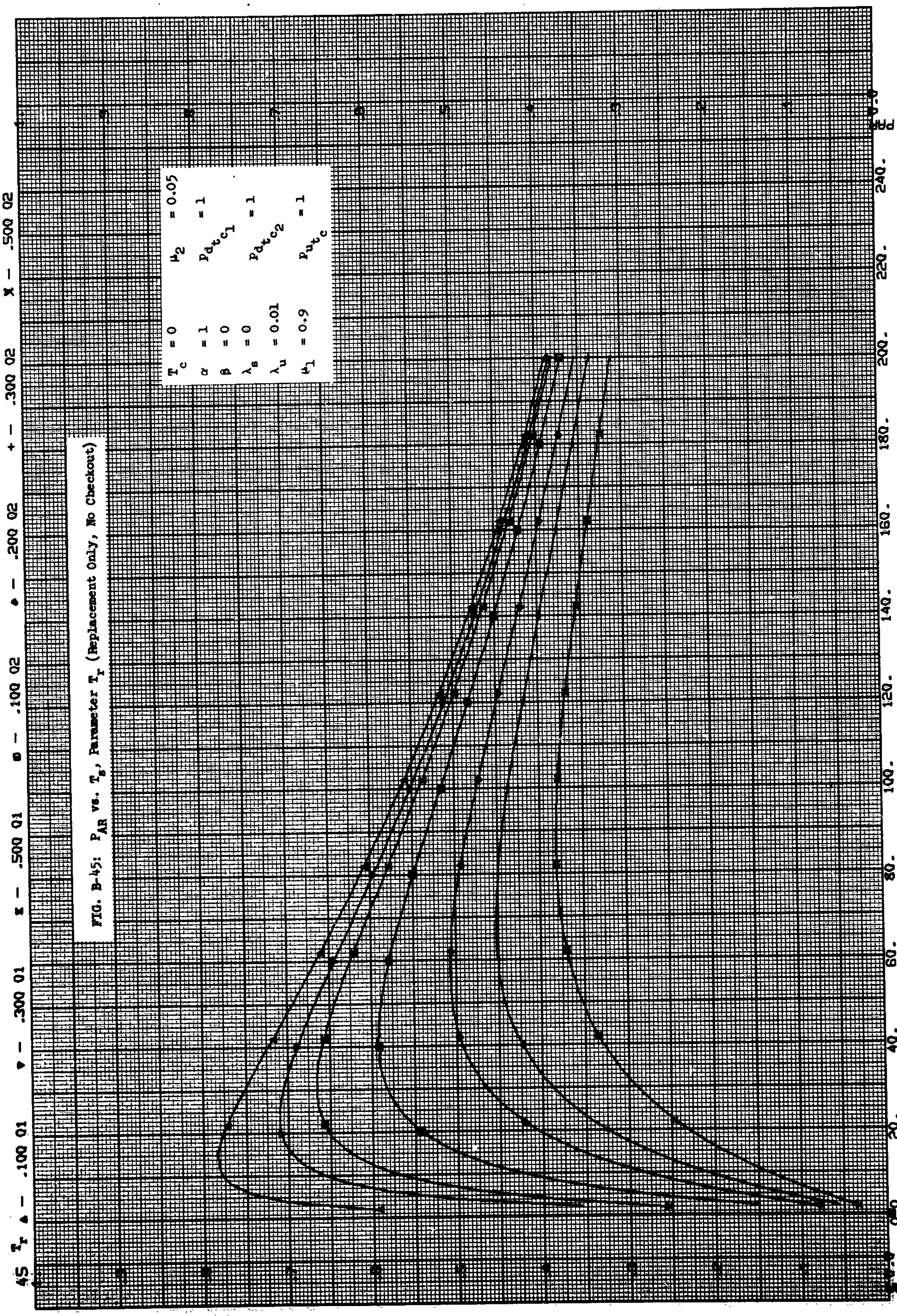

FIG. B-45: P_{AR} vs. T_s, Parameter T_r (Replacement Only, No Checkout)

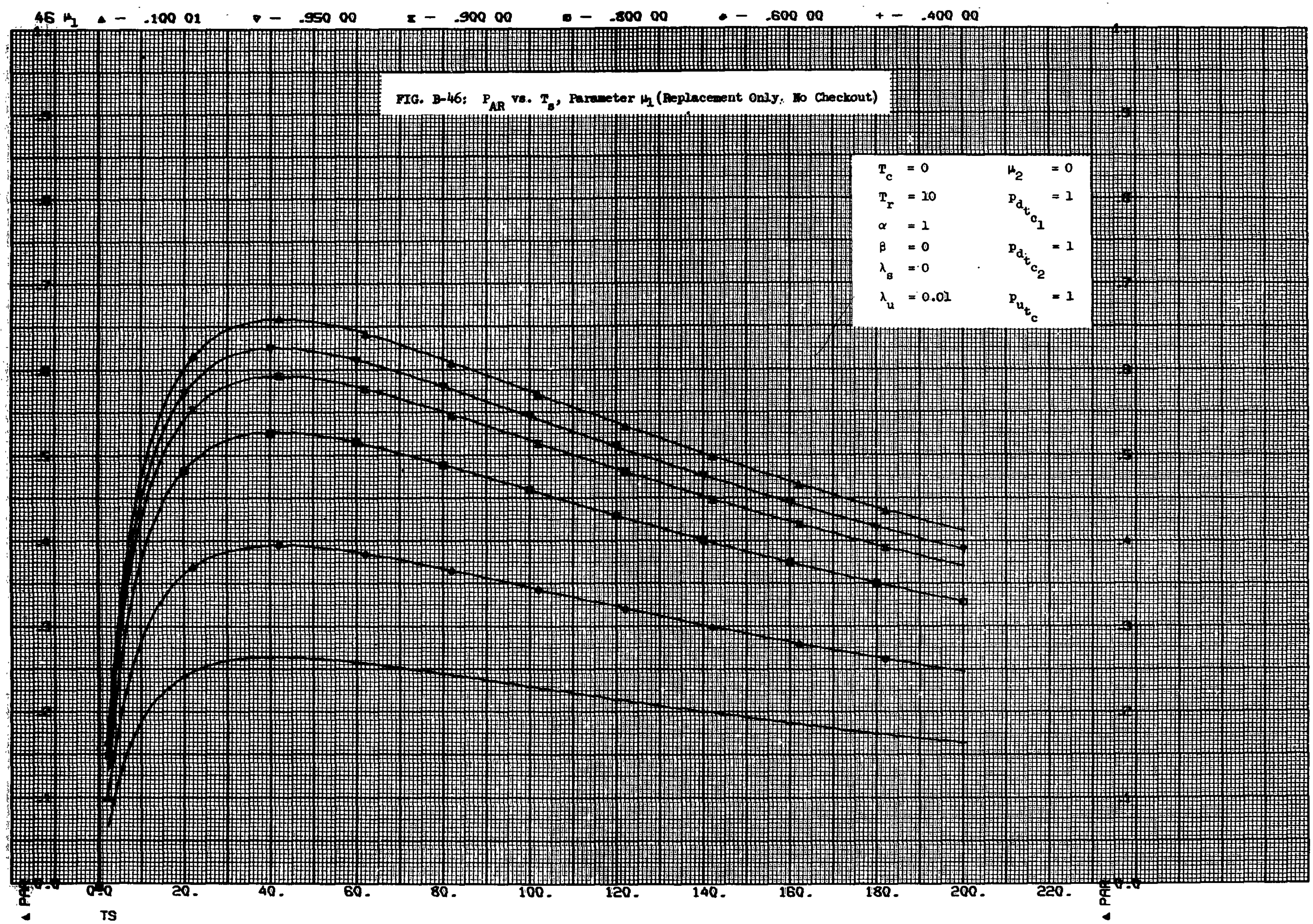

FIG. B-46: P_{AR} vs. T_s, Parameter μ_1 (Replacement Only, No Checkout)

Table B-1. List of Symbols Appearing in Appendix B

P_{AR}	=	Probability of alert readiness, or probability the system is in standby and non-failed at a random point in time
T_s	=	Duration of standby period
T_c	=	Duration of checkout period
T_r	=	Duration of repair period
α	=	Probability of calling a system with no detectable failures bad during checkout
β	=	Probability of calling a system with a detectable failure good during checkout
λ_s	=	Rate of occurrence of detectable failures during standby
λ_u	=	Rate of occurrence of undetectable failures during standby
μ_1	=	Probability a repaired system is unfailed
μ_2	=	Probability a repaired system is failed detectably
$1-\mu_1-\mu_2$	=	Probability a repaired system is failed undetectably
Pd_{tc_1}	=	Probability of no detectable failures occurring during first portion of checkout (prior to test decision)
Pd_{tc_2}	=	Probability of no detectable failures occurring during second portion of checkout (after test decision)
Pd_{tc}	=	Probability of no detectable failures occurring during checkout $= P_{d_{tc_1}} P_{d_{tc_2}}$
Pu_{tc}	=	Probability of no undetectable failures occurring during checkout

DISTRIBUTION

Col W. F. Stevens, AFSC, Andrews AFB, Maryland
Lt Col Gillespie, SAC HQ, Offutt, Neb.
Lt Col R. P. Gingland, BSMB
Lt Col R. M. Hunt, AFSC, Andrews AFB, Maryland
Lt Col N. L. Murray, Minuteman SPO
Lt Col C. E. Nelson, 1002nd Inspector General Group
Lt Col J. M. Ross, BSOSR (2)
Maj J. S. Skaggs, SAC HQ, Offut, Neb.
Maj J. R. Stimpson, BSMTP
Lt D. J. Stukel, SBNEWS, SBAMA (2)
Lt Col H. E. Walker, BSOC
K. George, 1st STRATAD, DORQ, VAFB

STL Library (2)

<u>DDC</u>
Defense Documentation Center (20)

UNCLASSIFIED

TRW Space Technology Laboratories, Inc., One Space Park . Redondo Beach, California
NUMERICAL ANALYSIS OF SYSTEM AVAILABILITY AND OF PARAMETER ESTIMATION METHODS
by W. J. Howard and N. Kaufman. August 1963 (Revised August 1, 1964), 107 pp. incl. illus.
(6101-8161-TU001)
(Contract AF 04(694)-524) Unclassified Report

This document describes two problem areas concerning systems subject to periodic checkout, which were investigated with the aid of a computer. The first part (Parameter Estimation) summarizes the results of a Monte Carlo analysis to determine the feasibility and accuracy of measuring system failure rates, checkout error probabilities, and repair effectiveness from the numbers of systems passing and failing in three consecutive checkouts. The second part (Availability Analysis) describes

UNCLASSIFIED

UNCLASSIFIED

TRW Space Technology Laboratories, Inc., One Space Park . Redondo Beach, California
NUMERICAL ANALYSIS OF SYSTEM AVAILABILITY AND OF PARAMETER ESTIMATION METHODS
by W. J. Howard and N. Kaufman. August 1963 (Revised August 1, 1964), 107 pp. incl. illus.
(6101-8161-TU001)
(Contract AF 04(694)-524) Unclassified Report

This document describes two problem areas concerning systems subject to periodic checkout, which were investigated with the aid of a computer. The first part (Parameter Estimation) summarizes the results of a Monte Carlo analysis to determine the feasibility and accuracy of measuring system failure rates, checkout error probabilities, and repair effectiveness from the numbers of systems passing and failing in three consecutive checkouts. The second part (Availability Analysis) describes

UNCLASSIFIED

UNCLASSIFIED

TRW Space Technology Laboratories, Inc., One Space Park . Redondo Beach, California
NUMERICAL ANALYSIS OF SYSTEM AVAILABILITY AND OF PARAMETER ESTIMATION METHODS
by W. J. Howard and N. Kaufman. August 1963 (Revised August 1, 1964), 107 pp. incl. illus.
(6101-8161-TU001)
(Contract AF 04(694)-524) Unclassified Report

This document describes two problem areas concerning systems subject to periodic checkout, which were investigated with the aid of a computer. The first part (Parameter Estimation) summarizes the results of a Monte Carlo analysis to determine the feasibility and accuracy of measuring system failure rates, checkout error probabilities, and repair effectiveness from the numbers of systems passing and failing in three consecutive checkouts. The second part (Availability Analysis) describes

UNCLASSIFIED

UNCLASSIFIED

TRW Space Technology Laboratories, Inc., One Space Park . Redondo Beach, California
NUMERICAL ANALYSIS OF SYSTEM AVAILABILITY AND OF PARAMETER ESTIMATION METHODS
by W. J. Howard and N. Kaufman. August 1963 (Revised August 1, 1964), 107 pp. incl. illus.
(6101-8161-TU001)
(Contract AF 04(694)-524) Unclassified Report

This document describes two problem areas concerning systems subject to periodic checkout, which were investigated with the aid of a computer. The first part (Parameter Estimation) summarizes the results of a Monte Carlo analysis to determine the feasibility and accuracy of measuring system failure rates, checkout error probabilities, and repair effectiveness from the numbers of systems passing and failing in three consecutive checkouts. The second part (Availability Analysis) describes

UNCLASSIFIED

UNCLASSIFIED

mainly through a series of graphs, the quantitative variation of system availability with a number of operational and maintenance parameters representing time durations of standby, checkout, and repair, failure rates during standby and checkout, repair effectiveness, decision errors during checkout, and test coverage.

UNCLASSIFIED

UNCLASSIFIED

mainly through a series of graphs, the quantitative variation of system availability with a number of operational and maintenance parameters representing time durations of standby, checkout, and repair, failure rates during standby and checkout, repair effectiveness, decision errors during checkout, and test coverage.

UNCLASSIFIED

UNCLASSIFIED

mainly through a series of graphs, the quantitative variation of system availability with a number of operational and maintenance parameters representing time durations of standby, checkout, and repair, failure rates during standby and checkout, repair effectiveness, decision errors during checkout, and test coverage.

UNCLASSIFIED

UNCLASSIFIED

mainly through a series of graphs, the quantitative variation of system availability with a number of operational and maintenance parameters representing time durations of standby, checkout, and repair, failure rates during standby and checkout, repair effectiveness, decision errors during checkout, and test coverage.

UNCLASSIFIED